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# VIRTUAL POLE METHOD - ALTERNATIVE METHOD FOR PROFILING RACK TOOLS

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## ABSTRACT

*This paper presents a variant for determining the enwrapping condition, in the case of rack gear type tools. This variant was called the "virtual pole method". The position of the virtual pole $P_v$ is defined by the movement angle of the blank together with the centrode associated with the generated profile. The virtual pole method allows translating in the centrodes' absolute movements, a point on the generated profile, on to the rack tool profile, thus determining the geometric locus representing the generating tool profile. The new method is easier to apply, while remaining scientifically rigorous. By applying the proposed algorithm, it is no longer necessary to write explicitly the relative movements between the tool and the piece, which simplifies the calculation process and eliminates some of the possibilities for errors.*

**KEYWORDS:** rack-gear profiling, virtual pole, enwrapping condition.

## 1. INTRODUCTION

The profiling of tools that generate by enwrapping, by the rolling method, is based on a well-defined algorithm which consists in attending the following steps:

- defining the generated profiles equations in the reference system of the blank in analytical form;
- determination of the absolute movements of the blank and the tool;
- determination of the relative movement of the blank toward the tool;
- determination of the trajectories equations described by the points that belong to the piece profile during its relative movement towards the tool;
- determination of the enwrapping condition which allows the selection among the points of the previously determined trajectories family, of those points that belong to the enveloping of this family, too;
- association of the enwrapping condition at the trajectories family equations, which allows to determine the parametric equations of the generating tool profile.

This paper proposes a new method for profiling rack tool, i.e., the "virtual pole method".

The new method starts from the "theorem of normals" (Willis theorem), according to which: "the profiles that transmit rotation movement between two parallel axes allow a common normal in the contact point, normal which passes through the gearing pole".

The case of generating with rack tool is a particular case of gearing between profiles that transmit a rotation movement, which is characterized by the fact that one of the centrodes radii associated with the two profiles has infinite value.

Thus, the respective centrode, which is a circle for a radius finite value, turns into a straight line and the rotation movement transmitted between the two profiles degenerates into a translation movement.

The new method is based on the determination of the enwrapping condition by finding a "virtual pole" that is nothing but the intersection point between a normal taken to the piece profile into a certain point of that profile and the centrode associated with the piece.

As is known, in the case of generating with rack tool the centrode of the piece is a circle having the radius equal to the rolling radius.

Applying the absolute movement to the piece, movement that it has during generation, is found the position where the normal, taken from the current point off the profile, passes through the gearing pole. This time it's about the real gearing pole, meaning the tangency point between the two centrodes.

The current point position, transposed into the tool reference system, reference system to which it was applied the absolute movement of the tool, will be the position of a point on the tool profile, which at that time is in contact with the generated profile.

It is worth pointing out that, in the fixed reference system, the same point belongs to the gearing line. So, after determining the enveloping condition, the common point of the three curves - the piece profile, the tool profile and the gearing curve - is determined by the simple coordinate changing between the references systems involved.

## 2. THE PROFILING ALGORITHM

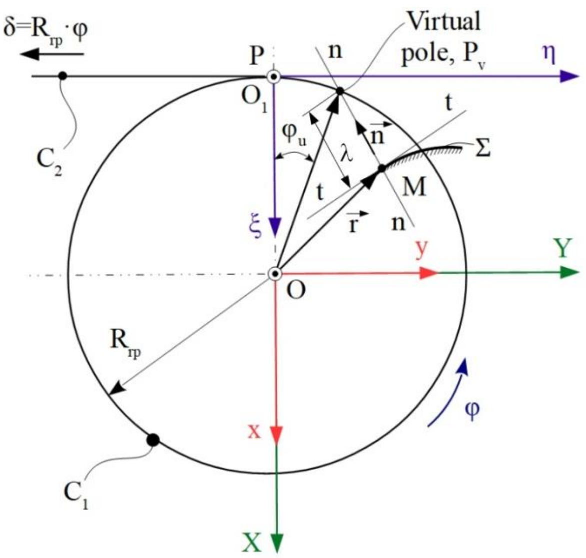


**Fig. 1.** *Rolling centrodes, $C_1$, $C_2$; reference systems; gearing pole, P and virtual pole, $P_v$, defined by the angle $\varphi_u$*

The rolling condition of the two centrodes: $C_1$ associated with the piece and $C_2$ associated with the future rack tool:

The profiling rack tool involves the use of three reference systems, see Figure 1.

The first is the fixed system, *xOy*, having the origin in the centre of the rolling circle of the piece.

The reference system associated with the generated profile is the *XOY* mobile system. This system initially has the axes superimposed on the fixed system axes, but during generation it rotates with the piece around the point *O*. The rotation angle of this system is notated with $\varphi$.

The reference system associated with the $C_2$ centrode is the mobile $\xi O_1 \eta$ system, joined with the tool. Initially, the *X*-axis is superimposed on the *X*-axis of the fixed system, but this axis translates during generation. The $\eta$-axis is parallel to the *y*-axis and always remains parallel to it. The distance between *y* and $\eta$ axes is the radius $R_{rp}$.

It is considered that the profile to be generated is known by the parametric equations:

$$C_{\Sigma y}\left|\begin{array}{l} X = X(u); \\ Y = Y(u), \end{array}\right. \qquad (1)$$

with *u* variable parameter.

The current point *M(X;Y)* is considered, whose position on the profile is determined by the value of the *u* parameter.

***Virtual pole***

The virtual pole is defined as the intersection point between the normal profile $\Sigma$ taken by the *M* point on the generated profile and the centrode associated with the piece - circle of radius $R_{rp}$.

In the rolling process of the two centrodes, with condition: $\delta = R_{rp}\cdot\varphi$, the virtual pole, $P_v$ will overlap for a certain value of the parameter with the gearing pole, *P* - the tangent point of $C_1$ and $C_2$ centrodes.

The angular position of the virtual pole $P_v$, in relation to the *P* pole, on the circle with radius $R_{rp}$ is defined as being the $\varphi_u$ angle.

The angle size, $\varphi_u$ - the virtual pole position - is determined from the intersection of the normal at $C_\Sigma$ - equations (1) with the rolling circle - the $C_1$ centrode - of equations (5).

The director parameters of the normal at the $\Sigma$ profile are described by the equation:

$$\vec{n}_\Sigma = \begin{vmatrix} \vec{i} & \vec{j} \\ \dot{X}_u & \dot{Y}_u \end{vmatrix} = \dot{Y}_u\cdot\vec{i} - \dot{X}_u\cdot\vec{j}. \qquad (2)$$

The normal vector at $\Sigma$, having the $\lambda$ length, is given by the equation:

$$\vec{N}_\Sigma = \lambda\cdot\vec{n}_\Sigma = \lambda\cdot\left(\dot{Y}_u\cdot\vec{i} - \dot{X}_u\cdot\vec{j}\right); \qquad (3)$$

$\lambda$ - variable scalar.

For a certain value of *u*, the normal at $\Sigma$ in the *M (X (u); Y (u))* current point has the equation:

$$\vec{N}_{XY} = \vec{r} + \vec{N}_\Sigma = \left[X(u) + \lambda\dot{Y}_u\right]\vec{i} + \left[Y(u) - \dot{X}_u\right]\vec{j}\ . \quad (4)$$

The $C_1$ centrode (circle of radius $R_{rp}$) has the parametric equations:

$$C_1 : \left|\begin{array}{l} X = -R_{rp}\cdot cos\,\varphi; \\ Y = R_{rp}\cdot sin\,\varphi, \end{array}\right. \qquad (5)$$

$\varphi$ - angular variable parameter.

The value of $\lambda$ for which the normal at the profile intersects the $C_1$ centrode can be determined if the intersection condition between $C_1$ and $N_\Sigma$ is established.

$$\begin{cases} X = -R_{rp}\cdot cos\,\varphi = X(u) + \lambda\cdot\dot{Y}_u; \\ Y = R_{rp}\cdot sin\,\varphi = Y(u) - \lambda\cdot\dot{X}_u. \end{cases} \qquad (6)$$

$$\lambda = \frac{-R_{rp}\cdot cos\,\varphi - X(u)}{\dot{Y}_u} = \frac{-R_{rp}\cdot sin\,\varphi + Y(u)}{\dot{X}_u}. \qquad (7)$$

From (7) results:

$$-\left[R_{rp}\cos\varphi + X(u)\right]\dot{X}_u + \left[R_{rp}\sin\varphi - Y(u)\right]\dot{Y}_u = 0 \quad (8)$$

$$-\left[X(u)\dot{X}_u + Y(u)\dot{Y}_u\right] = R_{rp}\left(\cos\varphi\dot{X}_u - \sin\varphi\dot{Y}_u\right) \quad (9)$$

It can be noted that the form (8) of the enwrapping condition is the same as the enveloping condition form determined according to the normals theorem.

Equation (9) allows determination of the $\varphi_u$ angle which defines the position of the virtual pole $P_v$ in the space *XY*.

Following the absolute movement of the piece with the $\varphi_u$ angle the virtual pole, $P_v$ overlaps the *P* pole. The *P* pole represents the tangency point of the two centrodes $C_1$ and $C_2$.

***The rack-gear profile***

In the $\varphi_u$ position, the *M*-point, belonging to the profile *Σ*, will also belong to the enwrapping profile *S* of the rack tool in the reference system *ξη*, moved according to its absolute movement.

The *M*-point position is obtained from the transformation given by the absolute movement of the *ξη* system, namely:

$$\xi = x + A; \quad A = \begin{pmatrix} R_{rp} \\ R_{rp}\cdot\varphi \end{pmatrix}. \quad (10)$$

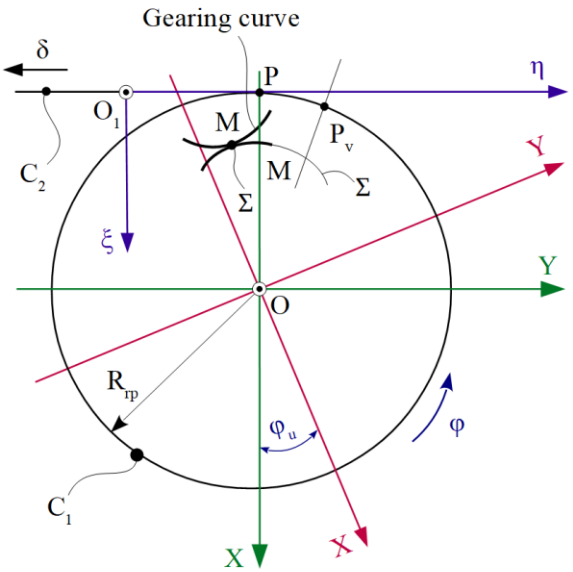


**Fig. 2.** *Transposition of the current point, M on the Σ profile on the gearing curve*

Note: The gearing curve, see Figure 2, determined as the geometric location of the tangency points between *Σ* and *S* in the space *xOy*, is given by the position of the *M*-point in the fixed system, respectively by the equation:

$$x = \omega_3^T(\varphi_u)\cdot X\ . \quad (11)$$

## 3. APPLICATION: RACK-GEAR FOR A SQUARE SHAFT

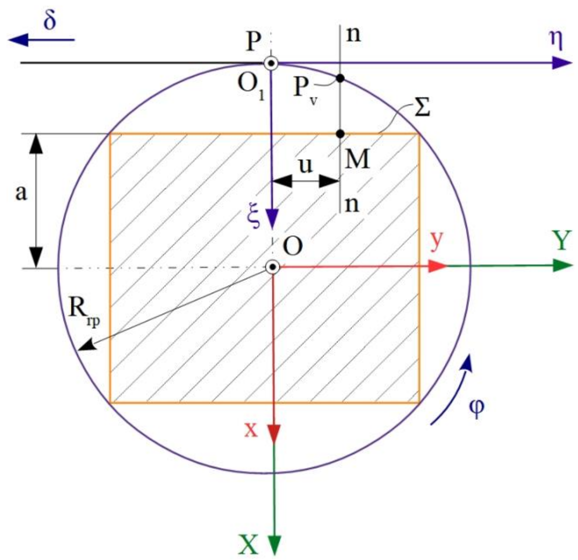


**Fig. 3.** *Square shaft. Reference systems and centrodes*

- Equations of the *Σ* profile, see Figure 3:

$$\Sigma \left| \begin{array}{l} X(u) = -a; \\ Y(u) = u. \end{array} \right. \quad (12)$$

- Enwrapping condition:

$$\begin{array}{l} \dot{X}_u = 0; \dot{Y}_u = 1; \\ \vec{N}_\Sigma = (-a + \lambda\cdot 1)\cdot\vec{i} + (u - 0)\cdot\vec{j}. \end{array} \quad (13)$$

$$C_1 \left| \begin{array}{l} X = -R_{rp}\cdot\cos\varphi; \\ Y = R_{rp}\cdot\sin\varphi. \end{array} \right. \quad (14)$$

$$\left.\begin{array}{l} -R_{rp}\cdot\cos\varphi = -a + \lambda \\ R_{rp}\cdot\sin\varphi = u; \end{array}\right\} \Rightarrow \left\{\begin{array}{l} \lambda = -R_{rp}\cdot\cos\varphi + a; \\ u = R_{rp}\cdot\sin\varphi. \end{array}\right. \quad (15)$$

$$\varphi_u = \arcsin\left(u/R_{rp}\right), \quad (16)$$

where $\varphi_u$ represents the position angle of the virtual pole on the piece centrode.

By rotating the *XOY* system around point *O*, the virtual point overlaps the *P* pole.

***Position on the gearing curve***

Gearing curve between the enwrapping profiles $C_{\Sigma_{XY}}$, of the generated blank and $S_{\xi\eta}$ of the generating rack-gear is defined as the geometric place of the contact points between two conjugated profiles in the fixed reference system - *xy*.

When the *XOY* system is rotated with the $\varphi_u$ angle, the position of the $M_{xy}$ point in the fixed system will be given by:

$$x = \omega_3^T(\varphi_u)\cdot X;$$
$$\begin{pmatrix} x \\ y \end{pmatrix} == \begin{pmatrix} -a\cdot cos\,\varphi_u - u\cdot sin\,\varphi_u \\ -a\cdot sin\,\varphi_u + u\cdot cos\,\varphi_u \end{pmatrix}. \qquad (17)$$

$$L_{C_\Sigma, S_\xi}\left| \begin{array}{l} x = -a\cdot cos\,\varphi_u - u\cdot sin\,\varphi_u; \\ y = -a\cdot sin\,\varphi_u + u\cdot cos\,\varphi_u. \end{array}\right. \qquad (18)$$

$M_{xy}$ point is a point on the gearing curve that is in contact with the $M_{XY}$ point on the $C_{\Sigma_{XY}}$ profile.

We observe that the $M_{xy}$ point on the $C_{\Sigma_{XY}}$ profile defined on the gearing curve represents in the *xy* system the transposed position of both the point on the $C_\Sigma$ profile and the future point belonging to the rack-gear profile $S_{\xi\eta}$ determined in the absolute movement of the generating rack-gear, see (19), (20) and (21).

The position of this point in the tool reference system is given by the absolute movement:

$$\xi = x + A. \qquad (19)$$

$$A = \begin{pmatrix} R_{rp} \\ R_{rp}\cdot\varphi_u \end{pmatrix}. \qquad (20)$$

$$M_{\xi\eta}\left| \begin{array}{l} \xi = -a\cdot cos\,\varphi_u - u\cdot sin\,\varphi_u + R_{rp}; \\ \eta = -a\cdot sin\,\varphi_u + u\cdot cos\,\varphi_u + Rrp\cdot\varphi_u. \end{array}\right. \qquad (21)$$

So, the tangent point coordinates between the tool profile and the piece profile will be given by the equations:

$$S\left| \begin{array}{l} \xi = -a\cdot cos\,\varphi_u - u\cdot sin\,\varphi_u + R_{rp}; \\ \eta = -a\cdot sin\,\varphi_u + u\cdot cos\,\varphi_u + Rrp\cdot\varphi_u; \\ \varphi_u = arcsin\left(u/R_{rp}\right). \end{array}\right. \qquad (22)$$

The generating tool profile was calculated for a shaft-type piece with a square section having the half side, $a = 14\ mm$, see Table 1.

**Table 1.** *Input data for the numerical application*

| a [mm] | $R_{rp}$ [mm] | $u_{min}$ [mm] | $u_{max}$ [mm] |
|---|---|---|---|
| 14.000 | 19.799 | -14.000 | 14.000 |

The numerical results are presented in Table 2, and the graphical representation of the rack-gear profile is shown in Figure 4.

It can be observed that the results obtained are identical to those obtained using established methods of profiling of tools that generate by enwrapping, by the rolling method (Gohman theorem or Normals theorem).

**Table 2.** *Coordinates of points on the tool profile*

| Crt. no. | ξ [mm] | η [mm] |
|---|---|---|
| **1** | 0.000 | -15.550 |
| ⋮ | ⋮ | ⋮ |
| **6** | 5.799 | 0.000 |
| ⋮ | ⋮ | ⋮ |
| **11** | 0.000 | 15.550 |

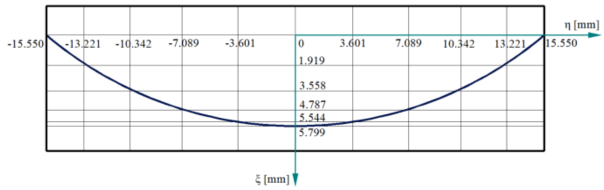


**Fig. 4.** *Rack-gear tool profile*

## 4. CONCLUSIONS

This paper presented a calculation variant of the enwrapping condition in the case of rack-gear type tools profiling, called the virtual pole method.

The “Virtual pole” is defined as the intersection point of normal at the $C_\Sigma$ profile of the generated blank with the centrode associated with the blank - circle of radius $R_{rp}$.

The position of the virtual point $P_v$ is defined by the angle of the blank movement on the circle of radius $R_{rp}$ - the $C_1$ circular centrode associated with the generated profile.

The Virtual point method at generating by enwrapping of some plane profiles associated with a centrodes rolling pair ($C_1$ - associated with the generated profile, circle of radius $R_{rp}$ and $C_2$ - straight line tangent to the $C_1$ centrode in the *P* point - gearing pole) allows transposition into the absolute movements of the $C_1$ and $C_2$ centrodes of certain point on the $C_\Sigma$ profile determined by a value of the *u* parameter from the associated *XY* system on the generating rack-gear tool profile in the $\xi\eta$ system thus determining the totality of *u* parameter values which respects the specific enveloping condition, the geometric place, which, in the $\xi\eta$ space, represents the profile of the generating rack-gear.

It should be noted that, through the new algorithm, the influence of relative movements remain, but it is eliminates the need to explicit write these movements and therefore the need to work with relatively complicated equations.

## REFERENCES

**[1] Litvin, F., L.,** *Theory of Gearing*, NASA Reference Publications 1212, AVSCOM Technical Report 88-C-135, 1989;

**[2] Radzevich, S.,** *Kinematic Geometry of Surface Machining*, CRC Press, Boca Raton, London, ISBN 978-1-4200-6340-0, 2007;

**[3] Oancea, N.,** *Surface Generation through Winding, Complementary Theorems, vol. 2*, “Dunarea de Jos” University Foundation Publishing House, ISBN 673-627-106-4, 2004;

**[4] Teodor, V.,** *Contributions to the elaboration of a method for profiling tools - Tools which generate by enwrapping*, Lambert Academic Publishing, ISBN 978-3-8433-8261-8, 2010.