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ROTOR BLADES MODELLING FOR A CENTRIFUGAL COMPRESSOR USING ANALYTICAL METHODS

Nicuşor BAROIU, Virgil Gabriel TEODOR,
Florin SUSAC, Nicolae OANCEA

Department of Manufacturing Engineering, "Dunărea de Jos" University of Galaţi,
florin.susac@ugal.ro

ABSTRACT

In this paper we propose an analytical method for rotor blade modelling for a centrifugal compressor. The considered rotor allows as hub surface a revolution surface with circular axial generatrix. The machining is made with a pre-formed tool with ball end. In this case, the contact with blade surface takes place both on the cylindrical and the spherical zone of the mill. A graphical solution developed in CATIA is presented, for a helix with constant axial pitch.

Keywords: centrifugal compressor, CATIA modelling, pre-formed mill

1. INTRODUCTION

Generating blade surfaces of a rotor belongs to a centrifugal compressor and it ? is included in the frame of free form surfaces machining [1] and may be regarded based on the fundamental theorems of the surfaces generating by wrapping [2].

The complexity of geometrical form for the centrifugal compressor as well as the diversity of applications for this compressor types in aerospace constructions, [3], [4], require CNC machine tools with five numerical controlled axis for machining.

This imposes the tool path planning at the machining of these surfaces types. This is a main concern in machining.

Many studies and proposals were made in order to increase the machining yield.

Also, it is noted that the general NC programs are ineffective for machining centrifugal pumps [5] and it is proposed a mathematical model for tool path using an interactive algorithm.

In order to improve the machining technology, Wu et al. propose an innovative approach which puts together the machining and the numerical simulation of the generation of centrifugal pumps blades [6]. In the same paper, it is proposed a rapid method for designing this type of blades. The method is based on the cubic spline curves and use machine tools with five numerical controlled axes.

There were studied and proposed methods to avoid collision between cutting tool and blank, determining the tool trajectory based on the blade and hub geometric model.

In this paper, we propose a method for blade generating, developed in analytic form, based on the generating trajectories, for machining a rotor of centrifugal compressor. It was developed a specific algorithm for a hub with circular generatrix. Also, it was developed, in CATIA, a graphical model of generating with end ball mill.

2. GENRATING KINEMATIX. REFERENCES SYTEMS

In what follows, it is studied the analytic modelling of blade generating for a hub with revolution body, with radius R_0, see figure 1.

In figure 1 are presented the reference systems associated with the rotor support and the cutting tool (end mill tool with composed profile: straight line and circular profile at tool end). The tool axis is inclined regarding the rotation axis and the tool end has radius r_0.

It is defined the radius of the axial hub profile, R_0. The circle's arc is defined between the normals tangent to the $\widehat{MN}$ arc, see figure 2. The points M and N represent the contact points of the $\widehat{MN}$ arc with the hub limit's, with radii R_b and R_t.

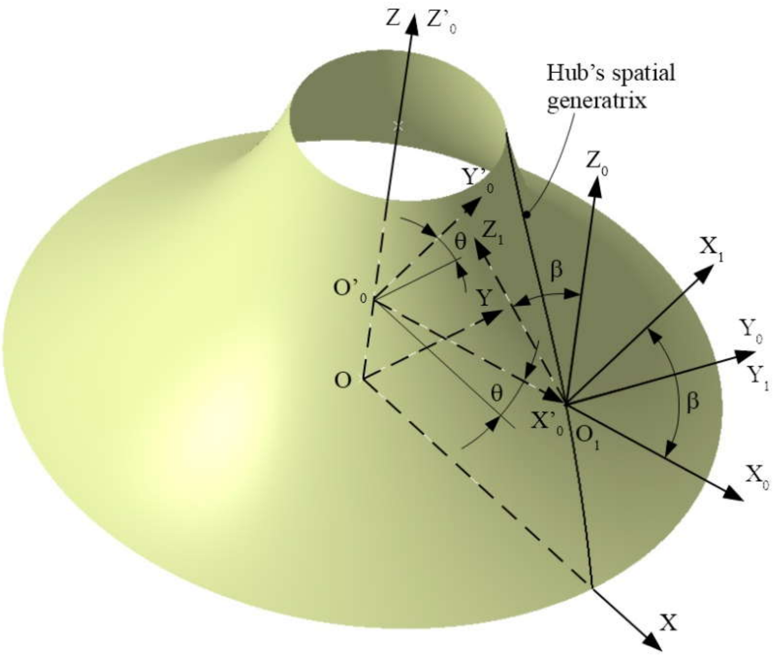

Fig. 1. *Hub with axial circular profile; reference systems*

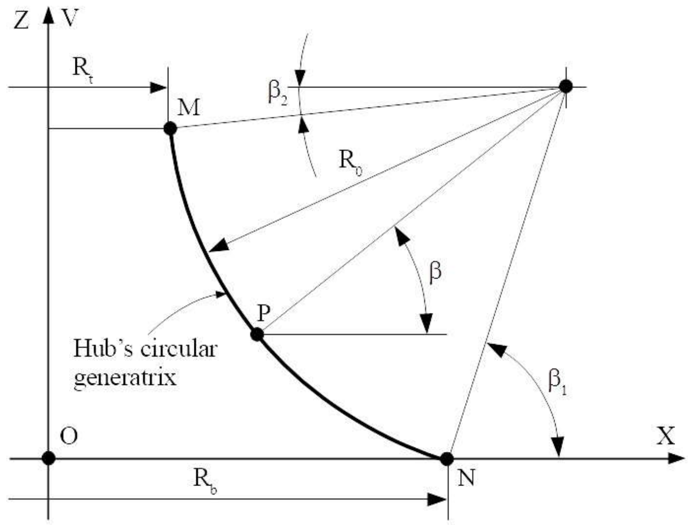

Fig. 2. *Axial hub's section*

The normals to the $\widehat{MN}$ arc, in *M* and *N* limits determine the centre of the circle's arc, point *O*.

The values of R_0, R_b and R_t radii are accepted as constructive values.

Are defined the following reference systems:

— *XYZ* global reference system, with *Z* axis joined with hub;

— $X_0Y_0Z_0$ mobile reference system, with origin in O_1, a current point onto the hub's circular generatrix;

— $X_0^{'}Y_0^{'}Z_0^{'}$ mobile reference system, initially overlapped to *XYZ*, joined with $X_0Y_0Z_0$ in the helical motion with $\vec{V}$ axis and *p* helical parameter. The $\vec{V}$ axis is overlapped to *Z* axis.

— $X_1Y_1Z_1$ reference system, with X_1 axis overlapped to peripheral surface of end mill tool.

The generating kinematics includes the rotation around the $\vec{V}(Z)$ axis, the movement *I*, linked with the translation movement *II*, along the $\vec{V}$ axis.

The rotation of the end mill tool around the $\vec{A}$ axis, *III* movement, is a cutting motion. The tool cutting edges belong to *S* surface, the primary peripheral surface of the end mill tool.

We have to notice that in the *II* motion, the *S* surface is self-generated. So, this motion does not affect the generating process. The helical surface generating is defined only by the assembly of motions *I* and *II*.

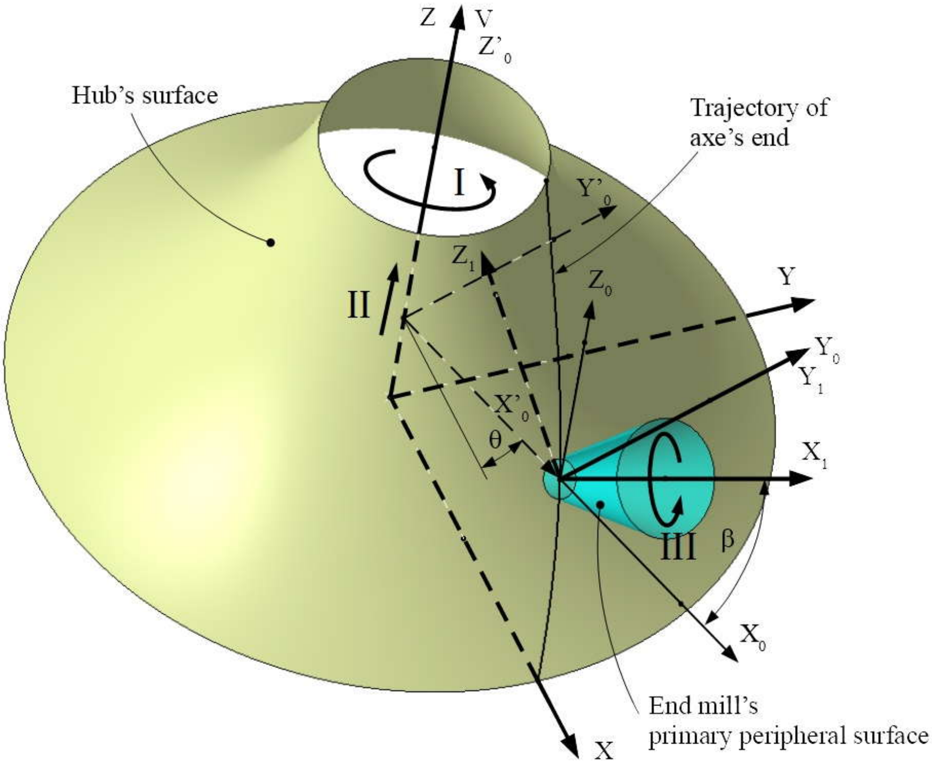

Fig. 3. *Generating kinematics*

Regarding the position of points *M* and *N* onto the axial generatrix of the hub, there are defined the angles β_1 and β_2, as angles between the normals to the *G* profile and *X* axis.

It is denoted with β the angle between the normal to the axial profile of the hub and the *X* axis, in the contact point of the end mill with the *G* generatrix, the point *P* (see figure 2).

3. COORDINATES TRANSFORMATIONS. ANALYTIC MODEL OF THE GENERATING TRAJECTORIES FAMILY

It is defined the relative position of the $X_1Y_1Z_1$ reference system regarding the $X_0Y_0Z_0$ axis, with X_1 axis of end mill:

$$X_0 = \omega_2(\beta)\cdot X_1, \tag{1}$$

where β is:

$$\omega_2(\beta) = \begin{pmatrix} \cos\beta & 0 & -\sin\beta \\ 0 & 1 & 0 \\ \sin\beta & 0 & \cos\beta \end{pmatrix}. \tag{2}$$

It is also defined the relative position of the $X_0^{'}Y_0^{'}Z_0^{'}$ and $X_0Y_0Z_0$ reference systems,

$$X_0^{'} = X_0 - A, \tag{3}$$

$$A = \begin{pmatrix} -R + R_0\cdot(\cos\beta - \cos\beta_1) \\ 0 \\ -p\cdot\theta \end{pmatrix}. \tag{4}$$

The link between the β angular parameter and the θ parameter is defined as:

$$\sin\beta = \frac{1}{R_0}\left(R_0\cdot\sin\beta_1 - p\cdot\theta\right). \tag{5}$$

The rotation motion around the $\vec{V}$ axis, with θ angular parameter, is given by:

$$X = \omega_3^T(\theta)\cdot X_0 \tag{6}$$

or, from (3),

$$X = \omega_3^T(\theta)\cdot\left[X_0 - A\right], \tag{7}$$

and, if consider equation (1), we obtain:

$$X = \omega_3^T(\theta)\cdot\left[\omega_2(\beta)\cdot X_1 - A\right], \tag{8}$$

representing the movement of $X_1Y_1Z_1$ space, joined with the end mill tool, regarding the *XYZ* space, joined with helix.

Developing we obtain the form:

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}\left[\begin{pmatrix} \cos\beta & 0 & -\sin\beta \\ 0 & 1 & 0 \\ \sin\beta & 0 & \cos\varphi \end{pmatrix}\cdot\begin{pmatrix} X_1 \\ Y_1 \\ Z_1 \end{pmatrix} - \begin{pmatrix} -R + R_0(\cos\beta - \cos\beta_1) \\ 0 \\ -p\cdot\theta \end{pmatrix}\right] \tag{9}$$

After developments, we reached the form, see (5):

$$\begin{aligned}X &= \left[X_1\cos\beta - Z_1\sin\beta + R - \right.\\ &\left.- R_0\left(\cos\beta - \cos\beta_1\right)\right]\cos\theta - Y_1\sin\theta;\\ Y &= \left[X_1\cos\beta - Z_1\sin\beta + R - \right.\\ &\left.- R_0\left(\cos\beta - \cos\beta_1\right)\right]\sin\theta + Y_1\cos\theta;\\ Z &= X_1\sin\beta + Z_1\cos\beta + R_0\cdot\left(\sin\beta_1 - \sin\beta\right).\end{aligned} \tag{10}$$

The equations (10) represent the generating trajectories family of points belonging to space $X_1Y_1Z_1$, joined with pre-formed tool, regarding the global reference system, joined with the helix.

If, from all the points belonging to the $X_1Y_1Z_1$ space, are selected those belonging to the peripheral primary surface geometric locus, so the relations (10) represent the generating trajectories family of *S* surfaces regarding the helix — the generated blade surface.

3.1. Pre-formed tool surface — S

In figure 4 it is presented the axial section of pre-formed tool and the associated reference system.

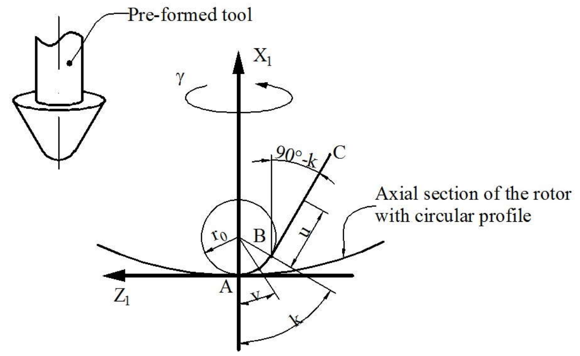

Fig. 4. *Axial tool's profile — S*

The axial profile of the pre-formed tool is composed from two filled elementary profiles:

— $\overset{\frown}{AB}$, arc with equations:

$$\overset{\frown}{AB}:\begin{vmatrix}X_1 = r_0 - r_0\cos v;\\ Y_1 = 0;\\ Z_1 = -r_0\sin v,\end{vmatrix} \tag{11}$$

$$v_{\min} = 0;\ v_{\max} = k. \tag{12}$$

Coordinates of *B* point:

$$\begin{aligned}X_{1B} &= r_0 - r_0\cos\kappa;\\ Y_{1B} &= 0;\\ Z_{1B} &= -r_0\sin\kappa.\end{aligned} \tag{13}$$

— $\overline{BC}$, straight line profile, with equations:

$$\overline{BC}:\begin{vmatrix}X_1 = r_0\left(1-\cos\kappa\right) + u\cdot\sin\kappa;\\ Y_1 = 0;\\ Z_1 = -r_0\sin\kappa - u\cos\kappa.\end{vmatrix} \tag{14}$$

Revolving the equation assembly (11) and (14) around the Z_1 axis, the parametrical equations of the tool primary peripheral surface are obtained.

For *BC* zone, the equations are obtained:

$$S:\begin{vmatrix}X_1 = r_0\left(1-\cos\kappa\right) + u\sin\kappa;\\ Y_1 = \left(r_0\sin\kappa + u\cos\kappa\right)\sin\gamma;\\ Z_1 = \left(-r_0\sin\kappa - u\cos\kappa\right)\cos\gamma,\end{vmatrix} \tag{15}$$

with γ angular parameter, at rotation around the Z_1 axis (the conic pre-formed tool's axis).

The equations assembly (10) and (15) represent the generating trajectory family for points belonging to *S* tool surface, regarding the *XYZ* reference system, joined with helix.

3.2. Enwrapping condition for trajectories family

The enwrapping of the trajectories family (10), (15) represents the generated blade surface, meaning the blade of the rotor.

The variables, in the equations (10) and (15) assembly are: θ —angular parameter for rotation around *Z* axis; γ — angular parameter for rotation around Z_1 axis; *u* — linear parameter for translation along the cone generatrix.

In this case, the enwrapping condition for generating trajectories family (10) and (15) is expressed in form:

$$\left(\vec{R}_\theta, \vec{R}_u, \vec{R}_\gamma\right) = 0. \tag{16}$$

where vectors $\vec{R}_\theta$, $\vec{R}_u$, $\vec{R}_\gamma$ are partial derivative from (10), (15):

$$\begin{aligned}\vec{R}_\theta &= \dot{X}_\theta\vec{i} + \dot{Y}_\theta\vec{j} + \dot{Z}_\theta\vec{k};\\ \vec{R}_u &= \dot{X}_u\vec{i} + \dot{Y}_u\vec{j} + \dot{Z}_u\vec{k};\\ \vec{R}_\gamma &= \dot{X}_\gamma\vec{i} + \dot{Y}_\gamma\vec{j} + \dot{Z}_\gamma\vec{k}.\end{aligned} \tag{17}$$

From (10) there are determined the partial derivatives:

$$\begin{aligned}\dot{X}_\theta &= \left(-X_1\sin\beta\frac{d\beta}{d\theta} - Z_1\cos\beta\frac{d\beta}{d\theta} + \right.\\ &\left.+R_0\sin\beta\frac{d\beta}{d\theta}\right)\cos\theta - \left[X_1\cos\beta - Z_1\sin\beta + \right.\\ &\left.+R - R_0\left(\cos\beta - \cos\beta_1\right)\right]\sin\theta - Y_1\cos\theta;\\ \dot{Y}_\theta &= \left(X_1\sin\beta\frac{d\beta}{d\theta} - Z_1\cos\beta\frac{d\beta}{d\theta} + \right.\\ &\left.+R\sin\beta\frac{d\beta}{d\theta}\right)\sin\theta + \left[X_1\cos\beta - Z_1\sin\beta + \right.\\ &\left.+R - R_0\left(\cos\beta - \cos\beta_1\right)\right]\cos\theta - Y_1\sin\theta;\\ \dot{Z}_\theta &= X_1\cos\beta\frac{d\beta}{d\theta} - Z_1\sin\beta\frac{d\beta}{d\theta} + R_0\cos\beta\frac{d\beta}{d\theta}.\end{aligned} \tag{18}$$

It can be observed that, from (5), results:

$$\beta = \arcsin\left(\sin\beta_1 - \frac{p}{R_0}\theta\right) \quad (19)$$

and, so,

$$\frac{d\beta}{d\theta} = \frac{\frac{-p}{R_0}}{\sqrt{1-\left(\sin\beta_1 - \frac{p}{R_0}\theta\right)^2}}. \quad (20)$$

As well, the partial derivatives are defined regarding the γ variable, see (15):

$$\begin{aligned} \dot{X}_\gamma &= \left(\dot{X}_{1_\gamma}\cos\beta - \dot{Z}_{1_\gamma}\sin\beta\right)\cos\theta - \dot{Y}_{1_\gamma}\sin\theta; \\ \dot{Y}_\gamma &= \left(\dot{X}_{1_\gamma}\cos\beta - \dot{Z}_{1_\gamma}\sin\beta\right)\sin\theta + \dot{Y}_{1_\gamma}\cos\theta; \quad (21) \\ \dot{Z}_\gamma &= \dot{X}_{1_\gamma}\sin\beta + \dot{Z}_{1_\gamma}\cos\beta, \end{aligned}$$

where, see (15), the partial derivatives are defined:

$$\begin{aligned} \dot{X}_{1_\gamma} &= 0; \\ \dot{Y}_{1_\gamma} &= \left(r_0\sin\kappa + u\cos\kappa\right)\cos\gamma; \quad (22) \\ \dot{Z}_{1_\gamma} &= \left(r_0\sin\kappa + u\cos\kappa\right)\sin\gamma. \end{aligned}$$

There are defined the partial derivatives regarding u parameter:

$$\begin{aligned} \dot{X}_u &= \left(\dot{X}_{1_u}\cos\beta - \dot{Z}_{1_u}\sin\beta\right)\cos\theta - \dot{Y}_{1_u}\sin\theta; \\ \dot{Y}_u &= \left(\dot{X}_{1_u}\cos\beta - \dot{Z}_{1_u}\sin\beta\right)\sin\theta + \dot{Y}_{1_u}\cos\theta; \quad (23) \\ \dot{Z}_{1_u} &= \dot{X}_{1_u}\sin\beta + \dot{Z}_{1_u}\cos\beta. \end{aligned}$$

$$\begin{aligned} \dot{X}_{1_u} &= \sin\kappa; \\ \dot{Y}_{1_u} &= -\cos\kappa\sin\gamma; \quad (24) \\ \dot{Z}_{1_u} &= -\cos\kappa\cos\gamma. \end{aligned}$$

3.3. The characteristic curve

The characteristic curve represents the contact curve between the S surface, (15), and the blade surface.

The characteristic in the generating process, in the XYZ reference system, is obtained by associating the trajectories family (10), the enwrapping condition.

In principle, the trajectories family has form:

$$\begin{aligned} X &= X(\theta,\gamma,u); \\ Y &= Y(\theta,\gamma,u); \quad (25) \\ Z &= Z(\theta,\gamma,u). \end{aligned}$$

The enwrapping condition is:

$$\left(\vec{R}_\theta, \vec{R}_\gamma, \vec{R}_u\right) = 0. \quad (26)$$

In addition, for the geometric locus of the contact points to be a curve, the θ parameter has to be constant:

$$\theta = \text{const.} \quad (27)$$

The (25), (26) and (27) equations assembly represents, in the XYZ reference system, the characteristic curve, the geometric locus of contact points between the S surface and Σ, surface of rotor's blade, joined with the hub, but not defined yet.

In this way, for $\theta = 0$, the assembly of partial derivatives is determined from (18):

$$\begin{aligned} \dot{X}_{\theta=0} &= -X_1\sin\beta\frac{d\beta}{d\theta} - Z_1\cos\beta\frac{d\beta}{d\theta} + \\ &+ R_0\sin\beta\frac{d\beta}{d\theta} - Y_1; \\ \dot{Y}_{\theta=0} &= X_1\cos\beta - Z_1\sin\beta + \quad (28) \\ &+ R - R_0\left(\cos\beta - \cos\beta_1\right); \\ \dot{Z}_{\theta=0} &= X_1\cos\beta\frac{d\beta}{d\theta} - Z_1\sin\beta\frac{d\beta}{d\theta} + R_0\cos\beta\frac{d\beta}{d\theta}. \end{aligned}$$

and $\left(\frac{d\beta}{d\theta}\right)_{\theta=0} = -\frac{p}{R_0\cos\beta_1}$.

Also, the partial derivatives regarding the U linear parameter, for θ=0, from (23):

$$\begin{aligned} \dot{X}_{u(\theta=0)} &= \dot{X}_{1u}\cos\beta_1 - \dot{Z}_{1u}\sin\beta_1; \\ \dot{Y}_{u(\theta=0)} &= \dot{Y}_{1u}; \quad (29) \\ \dot{Z}_{u(\theta=0)} &= \dot{X}_{1u}\sin\beta_1 + \dot{Z}_{1u}\cos\beta_1. \end{aligned}$$

Similarly, the partial derivatives are calculated regarding the γ angular parameter, for θ=0, from (21):

$$\begin{aligned} \dot{X}_{\gamma(\theta=0)} &= \dot{X}_{1\gamma}\cos\beta_1 - \dot{Z}_{1\gamma}\sin\beta_1; \\ \dot{Y}_{\gamma(\theta=0)} &= \dot{Y}_{1\gamma}; \quad (30) \\ \dot{Z}_{\gamma(\theta=0)} &= \dot{X}_{1\gamma}\sin\beta_1 + \dot{Z}_{1\gamma}\cos\beta_1. \end{aligned}$$

Now it is possible to write the enwrapping condition (26) as:

$$\begin{vmatrix} 0 & \sin\kappa & -Y_1 - \left(-X_1\sin\beta_1 - Z_1\cos\beta_1 + R_0\sin\beta_1\right)\cdot\frac{p}{R_0\cos\beta_1} \\ \left(r_0\sin\kappa + u\cos\kappa\right)\cos\gamma & -\cos\kappa\sin\gamma & X_1\cos\beta_1 - Z_1\sin\beta_1 + R \\ \left(r_0\sin\kappa + u\cos\kappa\right)\sin\gamma & -\cos\kappa\cos\gamma & p - \left(X_1\cos\beta_1 - Z_1\sin\beta_1\right)\cdot\frac{p}{R_0\cos\beta_1} \end{vmatrix} = 0. \quad (31)$$

The condition (31) can be rewrite in form:

$$\begin{vmatrix} 0 & -\tan\kappa & \left(X_1\sin\beta_1 + Z_1\cos\beta_1 - R_0\sin\beta_1\right)\cdot\dfrac{p}{R_0\cos\beta_1} \\ \cos\gamma & \sin\gamma & X_1\cos\beta_1 - Z_1\sin\beta_1 + R \\ \sin\gamma & \cos\gamma & p-\left(X_1\cos\beta_1 - Z_1\sin\beta_1\right)\cdot\dfrac{p}{R_0\cos\beta_1} \end{vmatrix} = 0. \tag{32}$$

Further, equation (32) and surface’s family (10), with condition θ=0, is rewritten as:

$$\begin{aligned} &X = X_1\cos\beta - Z_1\sin\beta + \\ &+\left[R - R_0\left(\cos\beta - \cos\beta_1\right)\right]; \\ &Y = Y_1; \\ &Z = X_1\sin\beta + Z_1\cos\beta, \end{aligned} \tag{33}$$

representing the characteristic curve on the blade’s flank.

In relations (32), (33), X_1, Y_1 and Z_1 have meanings give by relations (15).

Really, the (33) equations represent functions depending from parameters γ and *u*:

$$\begin{aligned} &X = X(\gamma,u); \\ &Y = Y(\gamma,u); \\ &Z = Z(\gamma,u). \end{aligned} \tag{34}$$

The (32) condition represents, in principle, an algebraic link between variables *u* and γ, in form

$$\gamma = \gamma(u). \tag{35}$$

Similarly, equations (15) and (32)assembly represent a spatial curve in the $X_1Y_1Z_1$ reference system, meaning the characteristic curve, identically to those from the blade flank.

Moving the characteristic curve:

$$\begin{aligned} &X_1 = X_1(u); \\ &Y_1 = Y_1(u); \\ &Z_1 = Z_1(u), \end{aligned} \tag{36}$$

with *u* variable parameter, with law (8),

$$X = \omega_3^T(\theta)\cdot\left[\omega_2(\beta)\cdot X_1 - A\right], \tag{37}$$

with $\omega_3^T(\theta) = \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$,

$\omega_2(\beta) = \begin{pmatrix} \cos\beta & 0 & -\sin\beta \\ 0 & 1 & 0 \\ \sin\beta & 0 & \cos\beta \end{pmatrix}$ and

$A = \begin{pmatrix} R + R_0\left(\cos\beta - \cos\beta_1\right) \\ 0 \\ p\theta \end{pmatrix}$.

The equation of the blade surface, in *XYZ* reference system, has parametrical form:

$$\Sigma \begin{vmatrix} X = X(u,\theta); \\ Y = Y(u,\theta); \\ Z = Z(u,\theta). \end{vmatrix} \tag{38}$$

The generated blade shape is highlighted in plane sections perpendicular to the hub axis, in form:

$$Z = H,\ H \text{— variable}, \tag{39}$$

$$Z(u,\theta) = H, \tag{40}$$

equivalent with a principle form

$$u = u(\theta). \tag{41}$$

In this way, the curves representing the generated blade profiles are defined by equations (matrix of coordinates):

$$\Sigma_H = \begin{pmatrix} X_1 & Y_1 & H \\ X_2 & Y_2 & H \\ \vdots & \vdots & \vdots \\ X_n & Y_n & H \end{pmatrix},\ H \text{ variable}. \tag{42}$$

4. NUMERICAL APPLICATION

It is presented a numerical application for a rotor with characteristics:

- top radius of the hub R_t = 30.111 mm;
- bottom radius of the hub R_b = 100 mm;
- radius of hub’s axial profile, see figure 2, R_0 = 150 mm;
- angle β_1 = 60°;
- angle β_2 = 15°;
- helical parameter *p* = 100 mm.

In order to graphically solve the problem, five crossing planes were generated, P_1… P_5. These are crossing planes of the hub, where were drawn the segments, L_1 ... L_5, with length corresponding to the radius of intersection circle between the spherical surface and the plan (see figure 5). The angle between these segments and the *X* axis corresponds with the value gives by the helical parameter and the crossing plane elevation, according to the equation:

$$\theta = \frac{180^\circ \cdot H}{p\cdot\pi}\ \text{[deg]}. \tag{43}$$

Between the ends of these segments is drawn a spline curve, *G*, which will represent the trajectory of the mill tool axis end in its relative motion regarding the conical surface.

It is generated a suit of planes, π_1 ... π_5, each π_i plane being defined by two lines. One of these lines is the *Z* axis, the other is the L_i line.

In each of the π_i plane, the spline curve is projected, obtaining the Γ in-plane curve, see figure 5.

The toll axis position, for each of the M_1 … M_5 points, is obtained as perpendicular to the projected curve and passing through the current point M_i, i = 1…5, see figure 4.

A new suite of planes is generated, P'_1 ... P'_5, perpendicular to the *G* spline, each P'_i plane passing through one end of the tool axis. In each of these planes is drawn a segment r_{vi}, perpendicular to the L_i corresponding segment and with length equals to the tool top radius.

It is drawn a spline curve which materializes the trajectory of the second end of the tool axis, the G_1 curve, see figure 5.

A new suite of five planes is generated, P''_1 … P''_5, perpendicular to the G_1 spline curve. Each P''_i plane passes through the point which corresponds to the second end of mill axis.

In each of these planes is drawn a segment r_{bi}, perpendicular to the L_i segment and with length equals to the tool bottom radius.

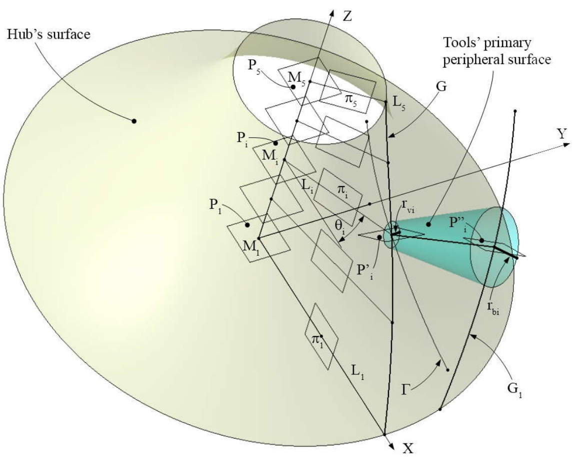

Fig. 5. *Construction of geometric features needed for graphical solving of problem*

There are drawn two spline curves, *G'*, passing through the ends of r_{vi} segments, and G'_1 passing through the ends of r_{bi} segments, i = 1…5.

A *sweep* surface is generated, with two guiding curves, namely both spline curves, G_1 and G'_1.

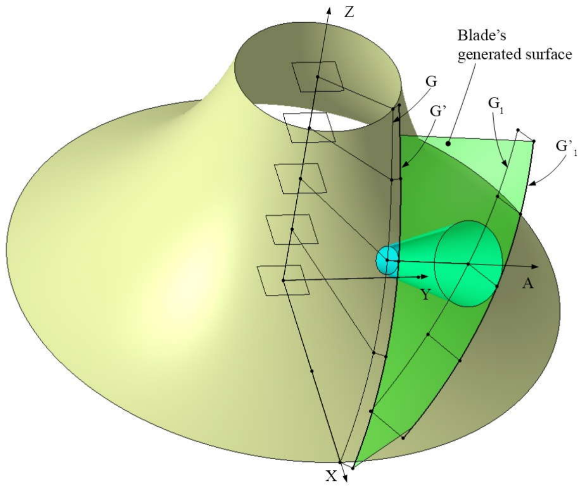

Fig. 6. *Rotor blade surface*

The sweep surface is intersected with planes $Z = H$.

In figure 7 and table 1 are presented crossing sections of the blade, in planes $Z = H$, with H variable.

Table 1. *Coordinates of points from the crossing sections of the blade [mm]*

H=0 [mm]			H=22.770 [mm]		
X	Y	Z	X	Y	Z
102.778	4.732	0.000	118.530	10.447	22.770
102.062	4.969	0.000	106.282	15.794	22.770
101.344	5.202	0.000	93.442	19.493	22.770
100.626	5.430	0.000	80.209	21.341	22.770
99.905	5.654	0.000	66.848	21.217	22.770
H=45.540 [mm]			H=68.311 [mm]		
X	Y	Z	X	Y	Z
117.719	26.675	45.540	77.830	56.005	68.311
99.687	33.572	45.540	63.194	52.941	68.311
80.566	36.157	45.540	49.518	46.880	68.311
61.396	33.971	45.540	37.177	38.415	68.311
43.303	27.252	45.540	26.238	28.192	68.311
H=91.081 [mm]					
X	Y	Z			
45.254	69.512	91.081			
43.129	67.651	91.081			
41.063	65.724	91.081			
39.057	63.735	91.081			
37.108	61.690	91.081			

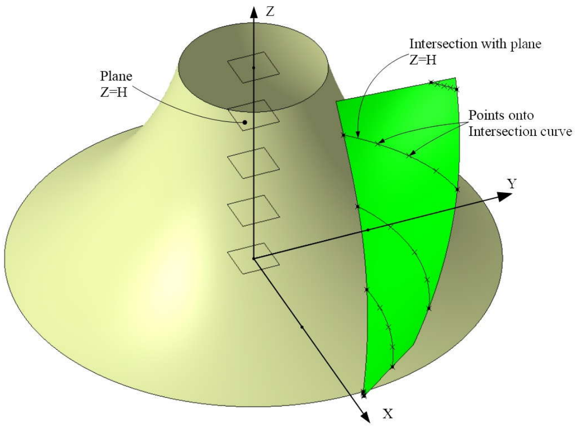

Fig. 7. *Crossing sections of the rotor blade; points onto the crossing sections*

6. CONCLUSIONS

This paper approaches in analytical form the issue of generating a blade of a rotor, which admits as hub a revolution surface with circular generatrix. The blade is generated with an end mill tool with a composed primary peripheral surface: a conical and a spherical surface.

The used method is the generating trajectories method, determining the composed surfaces family, in the reference system joined with hub.

The model of propeller blade surface is graphically generated, following the generating motion along the hub circular generatrix.

The methodology applied for the process is rigorous and easy to apply.

The presented methodology is applicable for complex shapes of hubs, too.

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