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ENWRAPPING SURFACES WITH POINT CONTACT — COMPARISSON BETWEEN CATIA METHOD AND ANALITICAL ONE

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ABSTRACT

In this paper, is made the comparison between the profiles of the worm tool, which generates a polygonal shaft, obtained by a classical analytical method and the graphical method developed in the CATIA design environment. The comparison is made in order to determine the quality of the profiling method of tools which generate, by point contact, ordered curls of surfaces.

The form of the intermediate surface (the generating rack-gear) and the form of characteristic curves are presented in graphical form, and the form of axial profiles of the worm mill calculated by the two methods is presented in numerical form.

KEYWORDS: worm mill, polygonal shaft, CATIA

1. Introduction

The goal of this paper is to determine the quality of de graphical method from the analytical methods in the case of profiling of tools which generate by enveloping, with point contact — the case of worm mill tool [1], [2], [5].

In the following is presented an application regarding the profiling of the primary peripheral surface of the worm tool reciprocally enveloping with a hexagonal shaft [4]. The profiling problem is solved by the two methods, the analytical one [3] and the graphical method developed in the CATIA design environment.

2. The worm mill tool's profiling. Generating kinematics

In Figure 1, it is presented the system of rolling axodes.

They are defined the reference systems:

xyz if the global reference system with *z* axis, the rotation axis of the axode joined with the curl of surfaces to be generated;

$x_0y_0z_0$ — global reference system with y_0 axis overlapped to the axis of the primary peripheral surface of the worm mill tool;

XYZ — relative reference system joined with the axode of the surface's curl to be generated;

$\xi\eta\zeta$ — relative reference system joined with the rack-gear axode (the in-plane surface overlapped with the plane $\eta\zeta$);

$X_1Y_1Z_1$ — relative reference system joined with the primary peripheral surface of the worm mill tool.

It is known the principle kinematics of the generating process:

$$x = \omega_3^T(\varphi_1)X, \tag{1}$$

representing the rotation of the blank's axode (revolution cylinder with R_{rp} radius), joined with *XYZ* reference system, with φ_1 angular movement parameter:

$$x = \xi + a;\ a = \begin{Vmatrix} -R_{rp} \\ -\lambda \\ 0 \end{Vmatrix}, \tag{2}$$

representing the rack-gear tool's axode translation (plane parallel with the $\eta\zeta$ plane), joined with the $\xi\eta\zeta$ reference system and λ movement parameter;

$$x_0 = \omega_2^T(\varphi_2)X_1, \tag{3}$$

the rotation of the $X_1Y_1Z_1$ reference system around the y_0 axis, with φ_2 angular movement parameter.

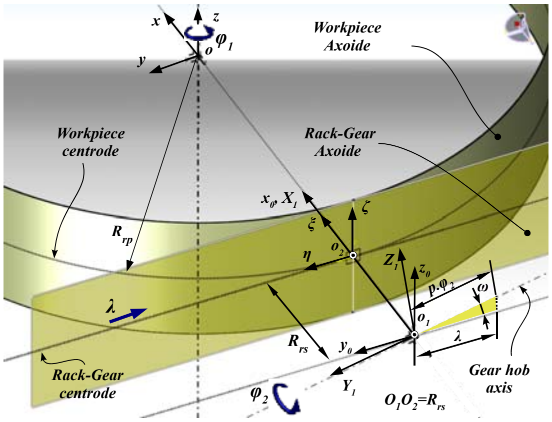

Fig. 1. *Reference systems and generating movements*

Also, are known the conditions:

$$\lambda = R_{rp} \cdot \varphi_1, \quad (4)$$

the rolling condition of the blank's centrode and the condition associated with the rack-gear;

$$\lambda = p \cdot \varphi_2 \cdot \cos\omega, \quad (5)$$

representing the dependency give by the worm mill tool's primary peripheral surface form (cylindrical worm with constant pitch and *p* helical parameter).

The relative motion of the reference system joined with the blank's axis (of the surface to be generated), *XYZ*, regarding the reference system associated with the rack-gear space, $\xi\eta\zeta$, is give by the transformation, see [6]:

$$\xi = \omega_3^T(\varphi_1) \cdot X - a, \quad (6)$$

with the definition of the current point from the surface to be generated, as a cylindrical surface with generatrix parallels with the direction $Z(\vec{k})$:

$$\Sigma \begin{vmatrix} X = X(u); \\ Y = Y(u); \\ Z = t, \end{vmatrix} \quad (7)$$

for *u* and *t* variables onto the surface to be generated.

The rack-gear surface determination

From (6) and (7), the Σ surfaces family is determined, in the rack-gear reference system, $\xi\eta\zeta$, with φ_1 variable parameter,

$$\begin{pmatrix} \xi \\ \eta \\ \zeta \end{pmatrix} = \begin{pmatrix} \cos\varphi_1 & -\sin\varphi_1 & 0 \\ \sin\varphi_1 & \cos\varphi_1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} X(u) \\ Y(u) \\ t \end{pmatrix} - \begin{pmatrix} -R_{rp} \\ -R_{rp} \cdot \varphi_1 \\ 0 \end{pmatrix}, \quad (8)$$

which it is associated the enveloping condition,

$$\left[X - X(u)\right] \cdot X'_u + \left[Y - Y(u)\right] \cdot Y'_u = 0 \quad (9)$$

where:

$$\begin{cases} X = R_{rp} \cdot \cos\varphi_1; \\ Y = R_{rp} \cdot \sin\varphi_1, \end{cases} \quad (10)$$

representing the "normals condition" with R_{rp} the radius of the rolling circle (the radius of cylindrical axode of the blank, which it is associated the curl of surfaces to be generated), see (9).

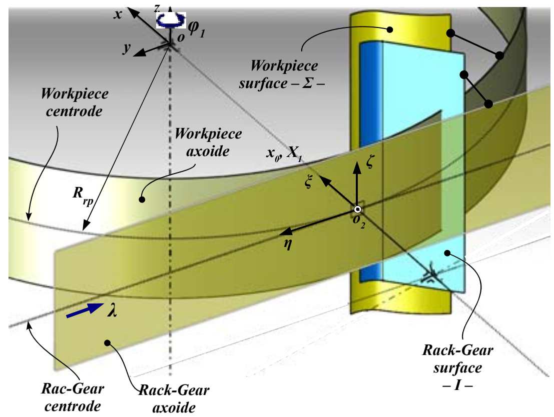

Fig. 2. *The Σ surface of the curl of surfaces to be generated, the rolling axodes, the rack-gear flank, I*

In principle, the enveloping of the surfaces family is described by the equations in form:

$$I \begin{vmatrix} \xi = \xi(\varphi_1, t); \\ \eta = \eta(\varphi_1 t); \\ \zeta = \zeta(\varphi_1, t), \end{vmatrix} \quad (11)$$

representing the form of the flank of rack-gear reciprocally enveloping with the curl of surfaces to be generated.

The worm mill tool's primary peripheral surface

Being known the surface of the rack-gear, the equations (11), it is proposed the determination of the characteristic curve (the contact curve) at the contact with the future primary peripheral surface of the worm mill tool, by using the method of helical movement decomposition [2], see Figure 3.

The helical motion which generate the worm mill tool's primary peripheral surface, $(\vec{V}, p)$, is decomposed in a sum of equivalent movements: the translation movement, along the $\vec{t}$ direction, of the cylindrical surface generatrix versor — the rack-gear flank — and a revolving motion around the $\vec{A}$, parallel with $\vec{V}$ (the axis of the worm mill tool’s helix) and at the distance:

$$a = p \cdot \tan\theta, \qquad (12)$$

from the axis of the helical surface, $\vec{V}$.

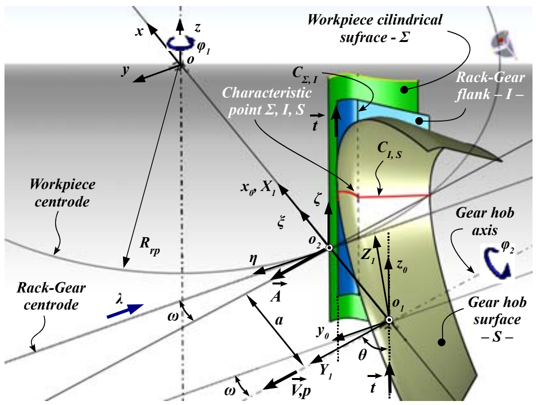

Fig. 3. *The helical movement decomposition – reference systems*

The value of the θ angular parameter is defined as the angle between the $\vec{t}$ versor and the helix axis $(\vec{V}, p)$.

In this way, the characteristic of *I* surface, in the motion composed by translation along the $\vec{t}$ generatrix and rotation around the $\vec{A}$ axis, not depend by the component of motion for which the surface is self-generated, being accomplished the identity,

$$\vec{N}_I \cdot \vec{t} \equiv 0 \qquad (13)$$

where, $\vec{N}_I$ is the normal at the *I* surface of the rack-gear flank (the cylindrical surface) is always perpendicularly to its own generatrix and, then, the condition for the determination of the characteristic curve in the helical motion $(\vec{V}, p)$ will depend only on the rotation motion around the $\vec{A}$ axis.

As follows, the characteristic curve of the *I* cylindrical surface — the rack-gear tool’s flank — in the helical motion with $\vec{V}$ axis and *p* helical parameter, is defined only as projection of the $\vec{A}$ axis on the *I* surface.

This is regarded as representing the geometrical locus of points belongs to the *I* cylindrical surface, for which the normals of this intersect the $\vec{A}$ axis.

They are defined; see Figure 3, the $\vec{A}$ axis, in the $x_0y_0z_0$ reference system,

$$\vec{A} = -\cos\omega \cdot \vec{j} + \sin\omega \cdot \vec{k}, \qquad (14)$$

and the normal at the *I* surface, not represented in Figure 3, in principle in form

$$\vec{N}_I = N_\xi \cdot \vec{i} + N_\eta \cdot \vec{j} + N_\zeta \cdot \vec{k} \qquad (15)$$

with N_ξ, N_η and N_ζ directrix parameters.

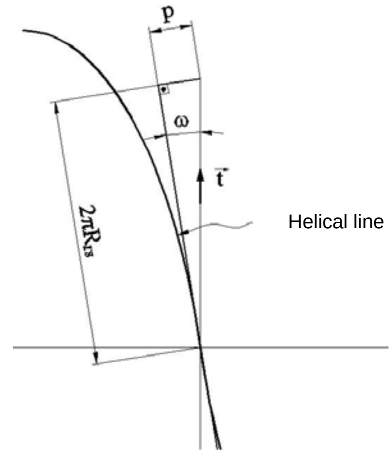

Fig. 4. *The helix of the cylinder with R_{rs} radius for the tool’s primary peripheral surface*

The value of the ω is determined from the condition that the helix belongs to the $(\vec{V}, p)$ helical surface on the cylinder with the radius R_{rs} to be parallel with the $\vec{t}$ versor of the rack-gear flank’s generatrix, Figure 4,

$$\tan\omega = \frac{2\pi p}{2\pi R_{rs}} = \frac{p}{R_{rs}} \qquad (16)$$

with *p* helical parameter of the worm tool’s primary peripheral surface.

So, the condition for the determination of the characteristic curve becomes:

$$(\vec{A}, \vec{N}_I, \vec{r}_I) = 0; \qquad (17)$$

where $\vec{r}_I$ is in form;

$$\vec{r}_I = [x_0(\varphi_1) - a] \cdot \vec{i} + y_0(\varphi_1) \cdot \vec{j} + t \cdot \vec{k}. \qquad (18)$$

In principle, the condition **Eroare! Fără sursă de referință.** represents a link between the variable parameters φ_1 and *t*, as:

$$q(\varphi_1, t) = 0. \qquad (19)$$

The assembly of (11) and (19) equations represent a geometrical locus onto the *I* surface, see Fig. 3, with significance of the *I* surface characteristic curve in the helical motion with $\vec{V}$ axis and *p* helical parameter — the axis and the helical parameter of the worm mill tool’s primary peripheral surface, *S*, reciprocally enveloping with the Σ surface to be generated.

The pairs of values for the parameters φ_1 and t, for which is accomplished the (19) condition, by replacement in the equation (11), determine the matrix

$$C_{I,S} = \left(\xi_i \quad \eta_i \quad \zeta_i\right),\ (i=1\ldots n), \tag{20}$$

representing the coordinates of the characteristic curve $C_{I,S}$.

The $C_{I,S}$ curve, known in numerical form, represents the tangency curve between the *I* surface — the rack-gear flank and the primary peripheral surface of the *S* helical tool, the worm mill tool which generate by enveloping the *Σ* profile.

It is obtained the form:

$$S\left|\begin{aligned}&X_1 = X_1\left(x_{0i}, y_{0i}, z_{0i}, \varphi_2\right);\\&Y_1 = Y_1\left(x_{0i}, y_{0i}, z_{0i}, \varphi_2\right);\\&Z_1 = Z_1\left(x_{0i}, y_{0i}, z_{0i}, \varphi_2\right),\end{aligned}\right. \tag{21}$$

representing the equations of the worm mill tool’s primary peripheral surface — the *S* surface.

Associating with the *S* surface the condition

$$Z_1 = 0, \tag{22}$$

is obtained the axial section of the worm mill tool, S_A, in principle, in form:

$$S_A\left|\begin{aligned}&X_1 = X_1\left(x_{0i}, y_{0i}, z_{0i}, \varphi_{2A}\right);\\&Y_1 = Y_1\left(x_{0i}, y_{0i}, z_{0i}, \varphi_{2A}\right),\end{aligned}\right. \tag{23}$$

$$i = 1 \ldots n,$$

with the φ_{2A} variable representing the value of the φ_2 parameter corresponding to the axial section, $Z_1 = 0$.

3. Numerical application

It was made the numerical application for a hexagonal shaft, with the geometry presented in the Figure 5. For this shaft we wish to determine the generating worm mill tool’s profile by the analytical method and by the graphical method.

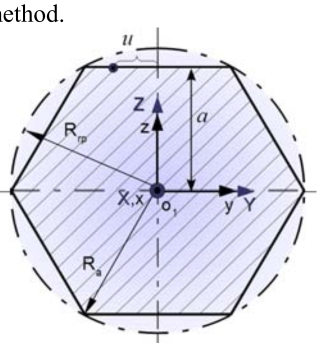

Fig. 5. *The profile of the hexagonal shaft*

The straight lined profile’s equations which represent the flank of the hexagonal shaft — the *Σ* profile, in this case are:

$$\Sigma\left|\begin{aligned}&X = -a_0;\\&Y = -u;\\&Z = t,\end{aligned}\right. \tag{24}$$

with *u* and *t* independent variables and

$$u \in \left[\frac{-R_{rp}}{2}, \frac{R_{rp}}{2}\right]. \tag{25}$$

The enveloping condition of the profiles family, see equations (7) and (8), is reduced at:

$$u = -R_{rp}\sin\varphi_1. \tag{26}$$

The equations assembly **Eroare! Fără sursă de referință.** and (26) determines the rack-gear tool’s profile:

$$I\left|\begin{aligned}&\xi = -a_0\cos\varphi_1 + R_{rp}\cos^2\varphi_1;\\&\eta = -a_0\sin\varphi_1 + R_{rp}\sin\varphi_1\cos\varphi_1 + R_{rp}\varphi_1;\\&\zeta = t.\end{aligned}\right. \tag{27}$$

The directrix parameters of the normal at the intermediary surface (27) are:

$$N_I\left|\begin{aligned}&N_\xi = a_0\cos\varphi_1 + 2R_{rp}\cos^2\varphi_1;\\&N_\eta = -a_0\sin\varphi_1 + 2R\sin\varphi_1\cos\varphi_1;\\&N_\zeta = 0.\end{aligned}\right. \tag{28}$$

In the same way, it is possible to write the enveloping condition **Eroare! Fără sursă de referință.**, where all the vectors were defined. Now, it is possible to determine the characteristic curve at the contact between the *I* rack-gear and the primary peripheral surface of the future worm mill tool *S*.

For the input data $R_{rp}=50$ mm; $R_a=50$ mm and $a=100$ mm, they are determined the profile of the worm mill axial section, in the $X_1Y_1Z_1$ reference system, see Table 1.

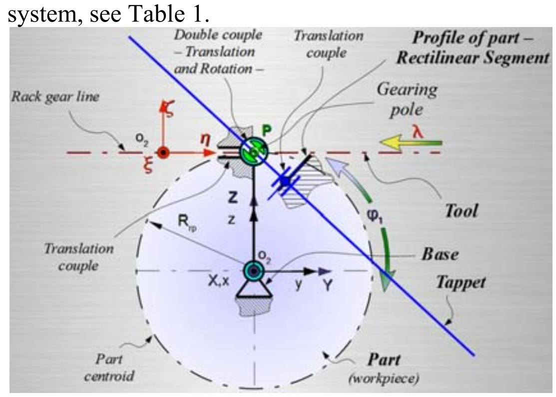

Fig. 6. *MGMC mechanism*

Kinematical method developed in the CATIA design environment

In the first stage, it is determined the rack-gear tool's profile, using an *MGMC* mechanism (see Figure 6) designed for a straight lined segment and composed from four *Part* elements, created in the CATIA design environment: *Base*, *Part*, *Tappet* and *Tool* (see Figure 7).

The *Base* file contain the fixed component of the mechanism and represent the guiding lines which couple the others elements. The hexagonal shaft is modelled in the *Part* file with the input data from the Table 1. The *Tool* is the file where is constructed the line which represent the rolling line of the reference rack-gear, *I*. The *Tappet* file is composed from two lines: *Tangent*, which represent the tangent which slide on the segment representing the hexagon side and *Normal*, which represent the normal draw from the start point (the side end), through the gearing pole.

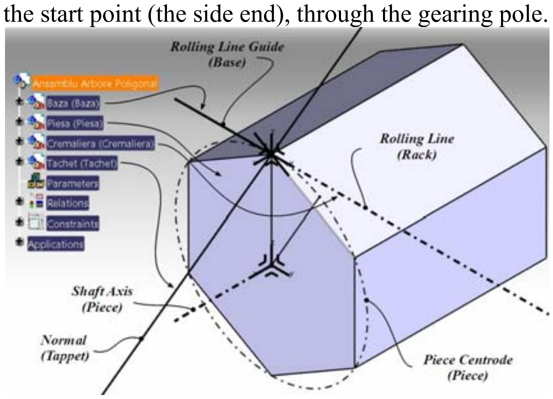

Fig. 7. *MGMC in the CATIA design environment — the mechanism elements*

These elements are assembled and constrained, making the kinematical couples which accomplish the rolling condition of the mechanism, the condition that the normal to be always in contact with the gearing pole.

In the *DMU Kinematics* design environment, the rack-gear's profile is determined as contact point between the normal and the hexagon side, in the rack-gear's reference system.

In the *Generative shape design*, it is obtained the reference rack-gear's primary peripheral surface, using the *Extrude* command, representing the virtual contact surface simultaneous enveloping with the surface of the hexagonal shaft and with the future worm mill tool, see Figure 8.

In this figure, it is represented the rack-gear, the characteristic curve at the contact of this with the worm mill tool's primary peripheral surface and the rack-gear crossing section.

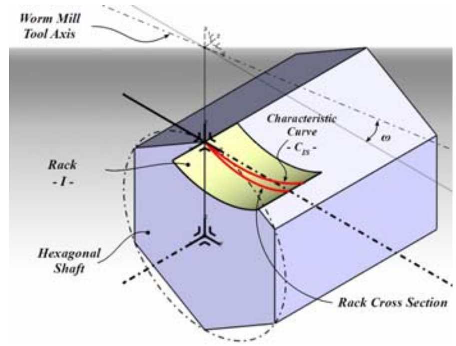

Fig. 8. *Characteristic curve between rack-gear and the worm mill primary peripheral surface*

The inclination angle of worm mill tool

It is established the inclination angle of the worm mill axis regarding the plane of the crossing section of the hexagonal shaft, *ω*, see Figure 9, knowing the relation

$$\tan\omega = \frac{p_e}{2\pi R_{rs}}, \tag{29}$$

where: p_e is the axial pitch of the helical surface — the worm mill tool's primary peripheral surface, *S*.
R_{rs} — rolling radius of the worm mill tool's primary peripheral surface with the reference rack-gear *I*.

It is known, as well as, the relation between the rack-gear pitch and the worm mill pitch, see Figure 9,

$$\cos\omega = \frac{p_{cr}}{p_e}. \tag{30}$$

In the considered case,

$$p_{cr} = \frac{2\pi R_{rp}}{6}. \tag{31}$$

Table 1. *Coordinates of the axial section profile*

Crt. no.	Graphical method			Analytical method		
	X_1 [mm]	Y_1 [mm]	Z_1 [mm]	X_1 [mm]	Y_1 [mm]	Z_1 [mm]
1	50.000	-26.551	0.000	50.001	-26.554	0.000
2	50.274	-26.064	0.000	50.274	-26.066	0.000
3	50.541	-25.572	0.000	50.541	-25.574	0.000
4	50.802	-25.078	0.000	50.802	-25.078	0.000
…	…	…	…	…	…	…
51	56.698	0.280	0.000	56.697	0.277	0.000
52	56.693	0.839	0.000	56.692	0.836	0.000
53	56.682	1.398	0.000	56.681	1.395	0.000
…	…	…	…	…	…	…
97	50.802	25.078	0.000	50.804	25.073	0.000
98	50.541	25.572	0.000	50.544	25.568	0.000
99	50.274	26.063	0.000	50.277	26.060	0.000

100	50.000	26.551	0.000	50.001	26.545	0.000

In figure 9, it is presented the projection of the straight line which cross the gearing pole, parallel with the worm axis and is inclined with the ω angle, projection which determine the on the reference rack-gear, the characteristic curve generating by enveloping the worm mill tool's primary peripheral surface, C_{IS}, see Figure 3.

In the Table 1 and Figure 10, they are presented the coordinates and form of the axial section profile for the worm mill tool, obtained by the two methods, the analytical one and the method developed in the CATIA design environment.

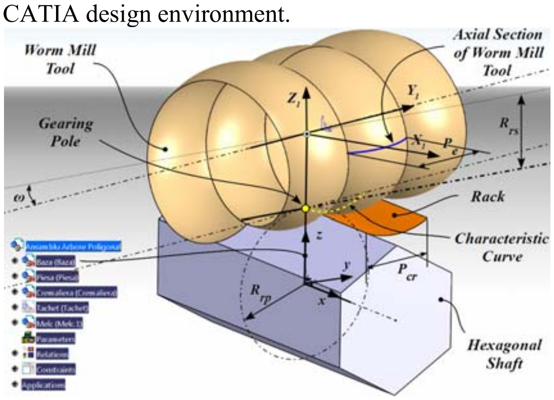

Fig. 9. *Projection line of the worm mill tool's axis*

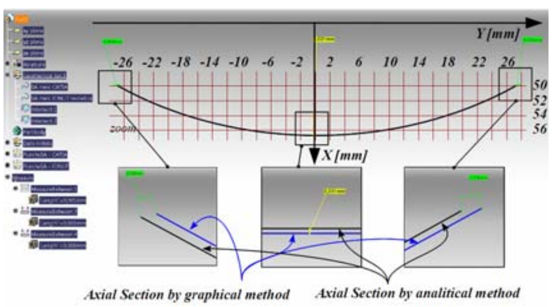

Fig. 10. *Axial section profile*

4. Conclusions

The determination of the axial section profile of the worm mill tool, which generates the hexagonal shaft, highlighted that the numerical results obtained by the two methods are identically.

The profile errors determined by the two methods are at level of 10^{-3} mm, which prove the fact the graphical method quality to rigorously solve the issue of the tools profiling, for tools which generate by enveloping with point contact.

The graphical method developed in the CATIA design environment is based on mechanism for virtual generation of the reference rack-gear profile, presented in this paper, as well as, on a method adequate to the capability of the CATIA for the determination of the characteristic curve, at the contact between a helical surface and a cylindrical one.

The graphical method has the advantage to highlight very easy the interference trajectories of the singular points from the enveloping profiles. The method allows the numerical interpretation of the results obtained by graphical modeling of the surfaces enveloping, in order to use it on CAD-CAI-CAM integrated systems.

AKNOWLEDGEMENTS

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Suprafețe în înfăşurare cu contact punctiform — comparație între metoda CATIA şi o metodă analitică

—Rezumat—

În lucrare, se prezintă o metodă dezvoltată în mediul de proiectare grafică CATIA, pentru profilarea suprafeței elicoidale reciproc înfăşurătoare cu un vârtej ordonat de suprafețe cilindrice.

Evidențierea calității metodei s-a făcut prin compararea rezultatelor numerice obținute pentru un caz concret — scula melc pentru generarea unui arbore hexagonal, în raport cu o metodă analitică.

Rezultatele numerice arată că, profilurile axiale ale sculelor melc obținute ca rezultat a aplicării celor două metode, grafică şi analitică, sunt identice din punctul de vedere al necesităților tehnice.

Metoda grafică descrisă în lucrare prezintă avantajul unei mai sugestive reprezentări a curbelor caracteristice şi a formei de ansamblu a suprafeței periferice primare a acesteia.