TOPOLOGICAL REPRESENTATION OF ROTATING CUTTER RECIPROCALLY ENVELOPING WITH A HELICAL SURFACE

S. Berbinschi, F.J. Santos Martin, N. Baroiu, V. Teodor

Dunarea de Jos University of Galati, Department of Manufacturing,
Robotics and Welding Engineering, Romania
silviu.berbinschi@ugal.ro

ABSTRACT

Generation of the helical surfaces by turning with rotating cutter tool is a process used in the mass machining. The process is made on specialized machines tool and is characterized by an increased productivity.

In this paper, is proposed a representation of the surfaces (profiles) in the CATIA design environment. A specific algorithm, developed in the graphical design environment allows determining the rotating cutter profile, reciprocally enveloping with the axial section of the helical surface to be generated.

It is developed an application for a cylindrical helical surface with constant pitch (an Arhimedes worm), representing the flanks of a trapezoidal thread. The profile of the rotary cutter is determined analytical and graphical.

KEYWORDS: topological representation, rotating cutter, CATIA

1. Introduction

The rotating cutters, as tools for surface generation, have limited applications, being used for the generation of long cylindrical helical surfaces (driving threads of machine tools) and, also, for generation of worms from the globoid gear.

The profiling method for the rotating cutter is based on the surfaces enveloping principles – the enveloping by the rolling method of the profiles associated with a couple of rolling centrodes [1], [2], [3], [5], [6].

The analytical methods lead to the determination of the rotating cutter teeth's flanks in a rigorous way. However, it is difficult to study by these analytical methods the surface's formation which determines the interference between the enveloping profiles – interference on 1st and 2nd degree.

A solution based on 3D modeling of the enveloping process, between the rotating cutter's profile and the axial section of the generated worm, was developed in the AutoCAD environment [4]. They are some limits of this method, due to the specific of the used algorithm.

In this paper is proposed a method, developed in the graphical design environment, which allows the 3D representation of the helical surface to be generated and following, the determination of the axial section of this. It was developed a specific algorithm, created for such application.

2. Generation process kinematics

In principle, the issue of the rotating cutter tool's profiling for generation of a cylindrical worm (cylindrical helical surface with constant pitch) may be reduced at an in-plane generation problem, in the axial section of the worm to be generate, see Figure 1.

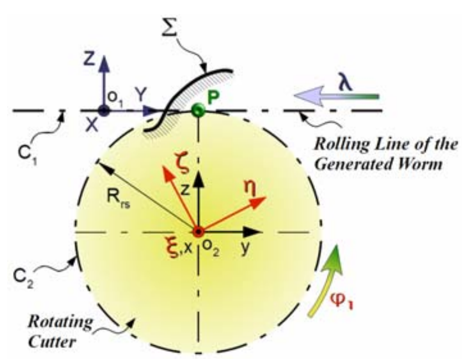

Fig. 1. *Rolling centrodes. Reference systems*

In Figure 1, they are presented:

C_1 is the straight lined centrode, associated with the axial section of the worm to generate Σ;

C_2 — circular centrode, with R_{rs} radius, associated with the rotating cutter;

xyz — global reference system, with *x* axis overlapped with rotation axis of the C_2 centrode;

XYZ — mobile reference system, joined with the C_1 centrode, where it is defined the profile of the axial section of the worm to be generate;

$\xi\eta\zeta$ — mobile reference system joined with the C_2 centrode of the rotating cutter.

It is defined the absolute motion of the rotating cutter and, linked with this, the motion of the mobile reference system $\xi\eta\zeta$:

$$x = \omega_1^T(\varphi_1)\cdot\xi \quad (1)$$

where

$$x = \begin{pmatrix} y \\ z \end{pmatrix} \text{ and } \xi = \begin{pmatrix} \eta \\ \zeta \end{pmatrix} \quad (2)$$

are the matrix associated with the current point of the *xyz* and respectively $\xi\eta\zeta$ spaces;

$$\omega_1(\varphi_1) = \begin{pmatrix} \cos\varphi_1 & -\sin\varphi_1 \\ \sin\varphi_1 & \cos\varphi_1 \end{pmatrix} \quad (3)$$

is the transformation matrix of rotation around the *x* axis.

Also, it is defined the translation movement of the axial section, of the C_1 centrode, along the *y* axis.

$$x = X + \begin{pmatrix} -R_{rs}\cdot\varphi_1 \\ R_{rs} \end{pmatrix} \quad (4)$$

which respect the rolling condition between the two centrodes, see Figure 1,

$$\lambda = R_{rs}\cdot\varphi_1, \quad (5)$$

with λ the velocity of the C_1 centrode (the *Y* axis).

Now, it is possible to define the relative motion between the relative reference systems,

$$\xi = \omega_1(\varphi_1)\cdot\left[X + \begin{pmatrix} -R_{rs}\cdot\varphi_1 \\ R_{rs} \end{pmatrix}\right] \quad (6)$$

For the axial section of worm to be generate is accepted the form

$$:\qquad \Sigma\left|\begin{aligned} &Y = Y(u); \\ &Z = Z(u). \end{aligned}\right. \quad (7)$$

In equation (7) was denoted with *u* the variable parameter along the curve which represents the axial section of the worm to be generated. From (6), it is possible to define the Σ profiles family, in the rotating cutter's reference system, $\xi\eta\zeta$.

$$\begin{pmatrix} \xi \\ \eta \\ \zeta \end{pmatrix} = \begin{pmatrix} \cos\varphi_1 & -\sin\varphi_1 \\ \sin\varphi_1 & \cos\varphi_1 \end{pmatrix}\cdot \left[\begin{pmatrix} Y(u) \\ Z(u) \end{pmatrix} + \begin{pmatrix} -R_{rs}\cdot\varphi_1 \\ R_{rs} \end{pmatrix}\right] \quad (8)$$

or, after developments,

$$(\Sigma)_{\varphi_1}\left|\begin{aligned} &\eta = \left[Y(u) - R_{rs}\cdot\varphi_1\right]\cdot\cos\varphi_1 + \\ &+\left[Z(u) + R_{rs}\right]\cdot\sin\varphi_1; \\ &\zeta = -\left[Y(u) - R_{rs}\cdot\varphi_1\right]\cdot\sin\varphi_1 + \\ &+\left[Z(u) + R_{rs}\right]\cdot\cos\varphi_1. \end{aligned}\right. \quad (9)$$

The enveloping of the $(\Sigma)_{\varphi_1}$ profile's family (9) represent, in the $\xi\eta\zeta$ reference system, the profile of the rotating cutter.

The enveloping is obtained, as it is known, associating with the equations (9), one of the following enveloping condition: Olivier; Gohman [1]; Willis [1], or a condition specifically for the complementary theorems: “minimum distance” [2]; “substitutive circles family” [2]; “in-plane generating trajectories” [3], see equation (11).

The gearing line

The gearing line is defined as the geometrical locus of contact points, in the global reference system, between the two conjugated profiles, of the blank, Σ, (7), and of the tool, rotating cutter, from the (9) curves family, together with the enveloping condition.

In this way, the (4) and (7) equations assembly determine the Σ profiles family, in the global reference system *xyz*:

$$\left(\Sigma_{\varphi_1}\right)_{xyz}\left|\begin{aligned} &y = Y(u) - R_{rs}\cdot\varphi_1; \\ &z = Z(u) + R_{rs}, \end{aligned}\right. \quad (10)$$

which, together with an enveloping condition, in form,

$$\eta'_u\cdot\zeta'_{\varphi_1} = \eta'_{\varphi_1}\cdot\zeta'_u, \quad (11)$$

determine, in the *xyz* global reference system, a geometrical locus which represents the gearing line.

The partial derivatives η'_u, η'_{φ_1}, ζ'_u and ζ'_{φ_1} are determine from equation (9).

2. The kinematical method in the CATIA design environment

In order to obtain the rotating cutter tool's profile, for a trapezoidal thread with the geometry presented in Figure 2, and the input parameters: $R_{rp}=24$ mm; $d=50$

mm; $d_1=46$ mm; $\alpha=15°$ and $r=0.6$ mm.

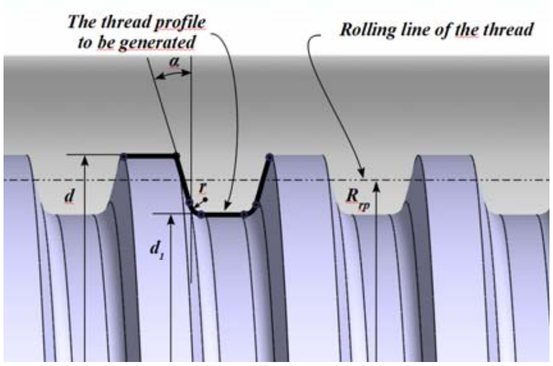

Fig. 2. *The axial section of the trapezoidal thread*

Generating mechanism in the CATIA design environment (M.G.M.C.)

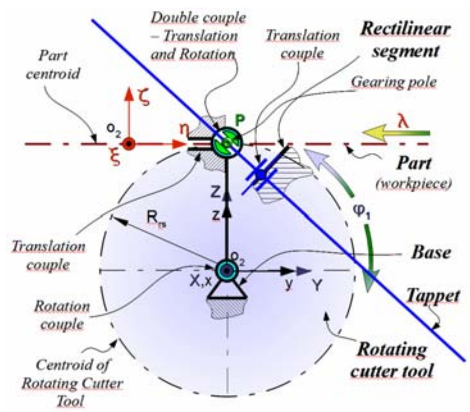

Fig. 3. *M.G.M.C. for straight line profile*

For the thread’s flank composed from two curve’s types, straight line segment and circle arc, they are designed specific generating mechanism in the CATIA design environment (M.G.M.C.).

Such mechanism is composed of the following elements, see Figures 3 and 4: Base; Piece; Tappet and Tool.

These elements are created in the *Part* and *Sketch* working environments and are assembled in the *Ansembly* environment, see Figure 5. In Figure 5, it is presented the thread’s solid model and the profile of the axial gap which will be generated with the rotating cutter tool.

In the *DMU Kinematics* working environment, are created the kinematics couples, the *Rack* couple, composed from the *Revolute* and *Prismatic* couples which allow the rolling between the two centrodes. The *Tappet* element represents, virtually, the normal at the profile of the thread axial section. The enveloping condition of the profiles is that the common normal at the piece and tool’s profile to cross the gearing pole (the tangency point of the two centrodes). This condition is accomplished by the couple *Prismatic*, between the *Tangent-Tappet* and the profile of thread to be generated.

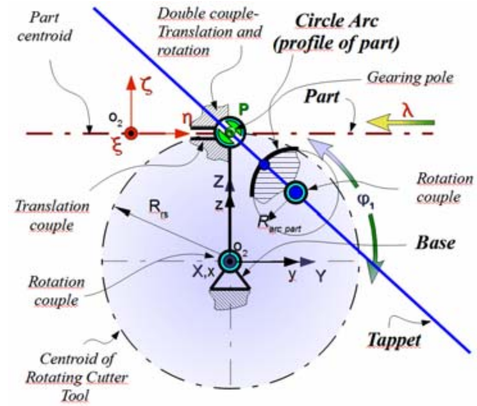

Fig. 4. *M.G.M.C. for circle arc profile*

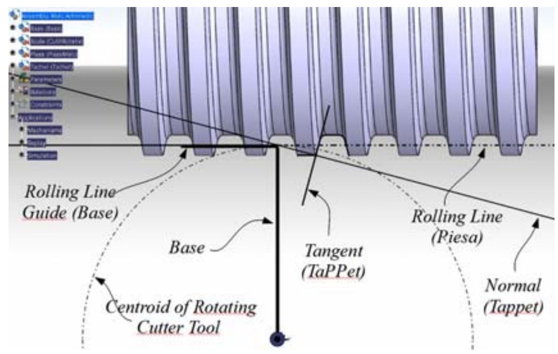

Fig. 5. *Part file for trapezoidal thread*

The driving element of the mechanism is the contact point between the *Tangent-Tappet* and the profile of the worm to be generated.

In Figure 6, it is presented the solid model of the rotating cutter tool, resulted by rolling the two *MGMC* mechanisms.

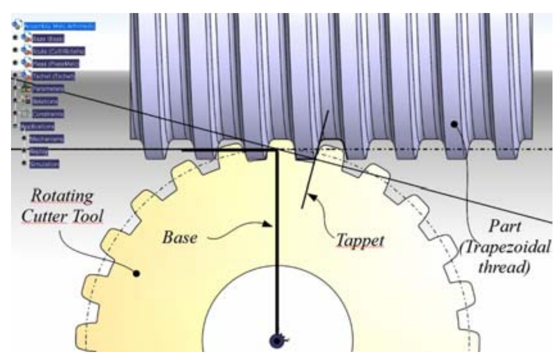

Fig. 6. *Solid model of the rotating cutter tool*

The final form of the rotating cutter profile is obtained by the elimination of the crossing curves generated by the singular points on the piece’s profile, see Table 1.

Table 1. *Coordinates of points on the tool’s profile*

AB - arc		BC – involute arc		CD – crossing curve	
η [mm]	ζ [mm]	η [mm]	ζ [mm]	η [mm]	ζ [mm]
4,326	0,185	3,732	1,000	3,732	-1,000
4,327	0,186	3,733	0,996	3,725	-1,026
…	…	…	…	…	…
4,910	0,391	4,301	-0,147	3,528	-1,345
4,918	0,389	4,313	-0,166	3,521	-1,345
4,925	0,387	4,326	-0,185	3,514	-1,344

The profile of the rotating cutter and the crossing curves were obtained by drawing the trajectories described in the tool’s reference system by the intersection point between the thread profile and the normal, respectively the singular points, see Figure 7.

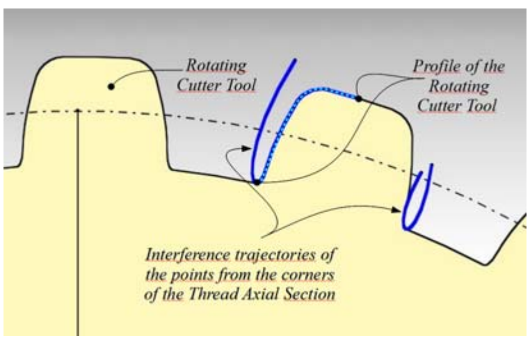

Fig. 7. *Crossing curves*

In Figure 7, they are highlighted the interference trajectories generated by the singular points from the axial section of the profile to be generated. By elimination of points which represent the interference trajectories result the rotating cutter profile.

The form and size of the crossing curves are influenced by the dimension and position of the centrode associated with the profile to be generated.

Conclusions

The kinematics method, in the CATIA design environment, has the quality to determine the form of the complex profile of the rotating cutter, with increased precision.

The graphical method allows the drawing of the interference trajectories and verification, from this point of view, of the size and position of the centrodes associated with enveloping profiles.

AKNOWLEDGEMENTS

The authors gratefully acknowledge the financial support of the Romanian Ministry of Education and Research through grant PN_II_ID_791/2008

REFERENCES

[1] Litvin, F.,L., Theory of Gearing, NASA RP- 1212(AVSCOM88-C035), Washinton DC, 1989.

[2] Oancea, N., Surface generation though winding, Volumes I & II Fundamental Theorems, “Dunarea de Jos” University Publishing House, ISBN 973-637-106-4, 2004.

[3] Teodor, V., Contribution to elaboration method for profiling tools which generate by enwrapping, Lambert Academic Publishing, ISBN 978-3-9433-8261-8, 2010.

[4] Baicu, I., Oancea, N., Profilarea sculelor prin modelare solida, Editura TEHNIC –INFO , Chisinau, ISBN 9975-63-172-X, 2002.

[5] Radzevich, S. P., Kinematic geometry of surface machining, CRC Press, Boca Raton, London, New York, ISBN 978-1-4200-6340-0.

[6] Rodin, P. R., Fundamentals of Theory of Cutting Tool Design, Mashgiz, Kiev, 1960.

Reprezentarea topologică a profilului cuțitului rotativ înfăşurător al unei suprafețe elicoidale

—Rezumat—

Generarea suprafețelor elicoidale prin strunjire cu scula de tip roată este un procedeu utilizat în fabricația în serie. Procesul se desfăşoară pe maşini-unelte specializate şi este caracterizat de o productivitate ridicată.

În lucrare, se propune o modalitate de reprezentare a suprafețelor (profilurilor) în mediul de proiectare grafică CATIA. Un algoritm specific, dezvoltat în mediul grafic, permite determinarea profilului cuțitului rotativ, reciproc înfăşurător secțiunii axiale a suprafeței elicoidale de generat.

Este dezvoltat un exemplu de aplicare pentru o suprafața elicoidala cilindrică de pas constant (melc arhimedic), reprezentând flancurile unui şurub trapezoidal. Se determină grafic şi numeric profilul cuțitului rotativ.