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NONCIRCULAR GEAR DESIGN AND GENERATION BY RACK CUTTER

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ABSTRACT

The paper considers the Gielis' superformula in the attempt of generalizing the procedure of planar noncircular gear centrode design. The superformula introduces six defining parameters that influence the curvature, magnitude and symmetry for closed/opened curves. The computer modeling of the gear centrode, based on direct profile method, enables the analysis of the defining parameters influence on centrode geometry and optimal parameter values to be selected. For convex shapes, the gear tooth profiles are further generated by simulating the gear cutting process, considering the given pitch line, a rack-cutter with standard parameters and the process specific kinematics.

KEYWORDS: non-circular gears, gear centrode, supershape, rack-cutter

1. Introduction

Since their appearance, noncircular gears have not enjoyed much confidence, mainly because of the difficulty in design and manufacture. If the first attempts of noncircular gears generation were made with devices that used a master noncircular gear and a master rack, later, thanks to the enveloping method, proposed by Litvin [1], conjugate tooth profiles of noncircular gears could be generated using tools (rack and shaper cutters) similar to those used in circular generation. This could be done by performing a pure rolling of the cutter's centrode along the noncircular gear's centrode. Reffering to the noncircular gear modeling process, once the mating centrodes are designed, the generation of teeth, along the centrodes, has proved to be a real challenge; to solve this difficult problem, many scientists have used different approaches to obtain proper teeth profiles.

Chang and Tsay obtained tooth profiles of noncircular gears, using the contact line method, considering both the cutting tool locus and the equation of meshing [2]. Danieli presented a method for determining the profile of noncircular gears, based on the integration of the meshing differential equation [3]. Bair developed a mathematical model of an elliptical gear tooth profile, proposing a generation mechanism, with hob or shaper cutter, and considering the theory of gearing [4]. Using the procedure proposed by Chang and Tsay, Figliolini and Angeles synthesized pairs of N-lobed elliptical gears and their racks [5].Tsay and Fong proposed a methodology to establish tooth profile of noncircular gears, based on the theory of gearing and the equation of meshing between the gear and a rack cutter [6]. Mundo also generated teeth profiles using a numerical approach, that integrates a differential equation describing the meshing process [7]. Litvinet al. used a matrix approach for the equation of meshing used to obtain the generated surfaces of elliptical gears by enveloping process [8]. Riaza et al. obtained the teeth profiles from the mathematical equation of the envelope curve and through a set of geometrical relations between the centrode of the rack cutter and the centrode of the noncircular gear [9]. Li et al. calculated the tooth profiles of noncircular gears using a method which reproduces the real gear shaping process, instead of deducing and solving complicated meshing equations [10]. In this approach, the tooth profile was obtained gradually, from the boundary that results by continuously plotting the shaper profile on the gear's transverse plane. Xia et al. obtained tooth profiles of bevel noncircular gears with intersecting axes, by means of geometry principles for spherical engagement [11]. Jing introduced a method that uses a pressure angle function in the angular displacement of gears for the description of the geometry and geometric characteristics of tooth profiles [12].

In the attempt of generalizing the noncircular gear design, the paper analyses the opportunity of using the Gielis’ supershape for the gear centrode modelling. As the supershape geometry is influenced by six defining parameters, a selection of their optimal values is required in order to further develop a first approach of the noncircular gear generation, i. e. the simulation of the gear manufacture by a rack-cutter.

2. Using supershape for noncircular gear centrode modeling

The theory of noncircular centrodes is developed on the basis of geometric and kinematic criteria of two rigid bodies engaged in conjugate rolling. Two hypothesis are considered to design a pair of mating centrodes, i. e. the desired transmission function and the desired driving centrode geometry, respectively. The approach of pre-designed centrode, known as Generating Profile Method, uses equation of traditional or modified ellipse [1, 4], Fourier series [6] and different specific monotonically increasing functions [13]. In the attempt of generalizing the centrode modelling process, the paper examines the opportunity of using a “superformula” to express the noncircular gear pitch curve geometry.

In 1996, Gielis proposed a generalized version of the superellipse polar equation by introducing different exponents for the traditional sin/cos terms and dividing the plane into sectors through a fractional argument of trigonometric functions. Hence, complex and spectacular shapes are generated, most of them being found in nature and industry [14, 15]. The Gielis defined the supershape as follows:

$$r(\emptyset) = \left[\left|\frac{1}{a}\cdot \cos\frac{n\emptyset}{4}\right|^{n_2} + \left|\frac{1}{b}\cdot \sin\frac{n\emptyset}{4}\right|^{n_3}\right]^{-\frac{1}{n_1}} \qquad (1)$$

where a, b are the semilengths of the major axes of the fundamental ellipse, therefore defining the size of the shape; n - a real number that determines the number of lobes of the shape and introduces its rotational symmetry; n_1, n_2 and n_3 - real numbers that lead to pinched, bloated or polygonal, symmetric or asymetric forms, depending on their values and relationship.

Tables 1-2 illustrate supershapes generated by arbitrarily chosen parameters. It can be seen that for circular shapes (tab. 1), the equality of the n_2 and n_3 parameters leads to n-lobed symmetrical centrodes. As long as $n_2 = n_3 < 1$, there are pinched forms, obviously unacceptable for a gear centrode configuration; while the exponents get close to 2, the curve gets bloated forms; increasing the exponents, concave-convex parts are inserted. If $n_2 \neq n_3$, a similar behaviour is noticed; low values of one of the exponents induce pinched vertex, corresponding to the major axis whose associated trigonometric term is affected by the exponent. Values higher than 2 are recommended for the exponents n_2 and n_3 in order to generate a proper centrode. In case of elliptical shapes (tab. 2), it is noticed that odd values of the n parameter lead to open curves. The rise of n_1 exponent limits the supershape geometry variation and enhances the curvature radii.

Table 1. *Supershapes with equal axis, a = b = 1*

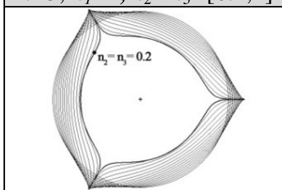
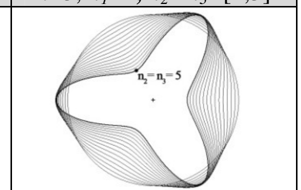
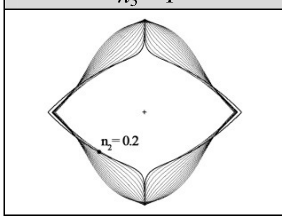
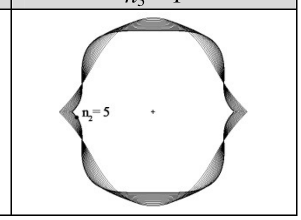
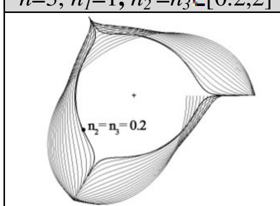
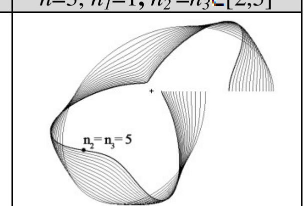
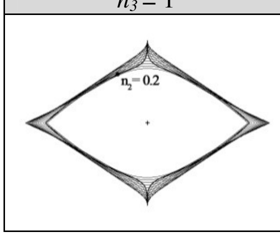
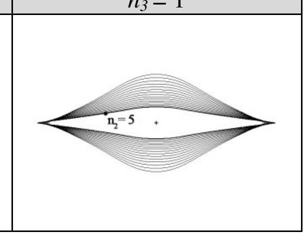
$n_2 = n_3$	
n=3, n_1=1, n_2=n_3∈[0.2,2]	n=3, n_1=1, n_2=n_3∈[2,5]
	
$n_2 \neq n_3$	
n=4, n_1=1, n_2 ∈[0.2,2], n_3 = 1	n=4, n_1=1, n_2 ∈[2,5], n_3 = 1
	

Table 2. Supershapes with a = 1.5, b = 1

$n_2 = n_3$	
n=3, n_1=1, n_2=n_3∈[0.2,2]	n=3, n_1=1, n_2=n_3∈[2,5]
	
$n_2 \neq n_3$	
n=4, n_1=1, n_2 ∈[0.2,2], n_3 = 1	n=4, n_1=1, n_2 ∈[2,5], n_3 = 1
	

3. Influence of supershape parameters on centrode geometry

As the paper is focused on the noncircular gear generation by rack-cutter, the analysis of the centrode curvature variation will be related to a standard rack of modulus m and profile angle α_c. In order to avoid undercutting, the minimal curvature radius of the centrode, ρ_{min}, should satisfy the inequality [16]:

$$\frac{\rho_{min}}{m} \geq \frac{1}{\sin^2\alpha_c} \quad (3)$$

The influence of the supershape parameters on the centrode minimal curvature radius, as well as the drawings of centrodes, for optimally chosen defining parameters, are illustrated in Tables 3-4. The minimal curvature radius is expressed using the dimensionless parameter $\overline{\rho_{min}} = \rho_{min}/m$.

Table 3. *Variation of centrode minimal curvature radius and geometry for a = b*

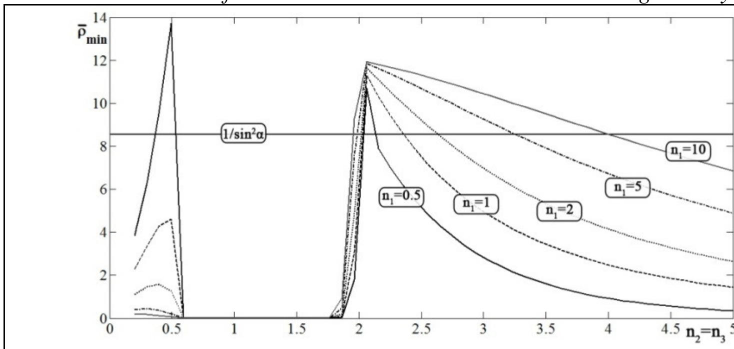
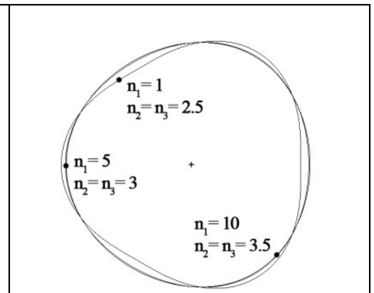
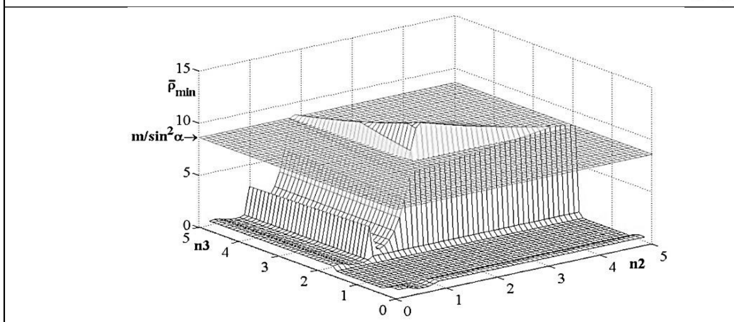
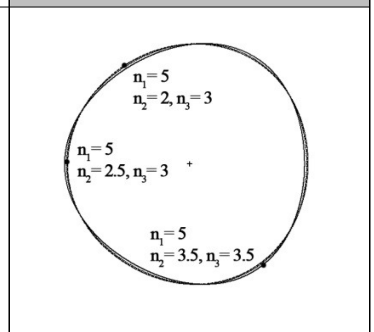
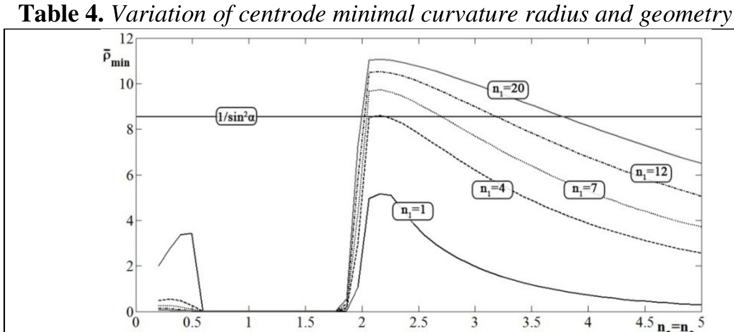
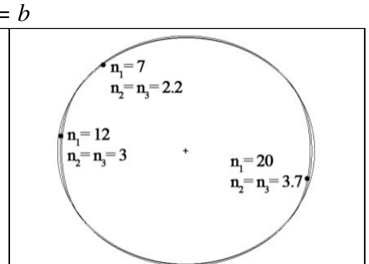
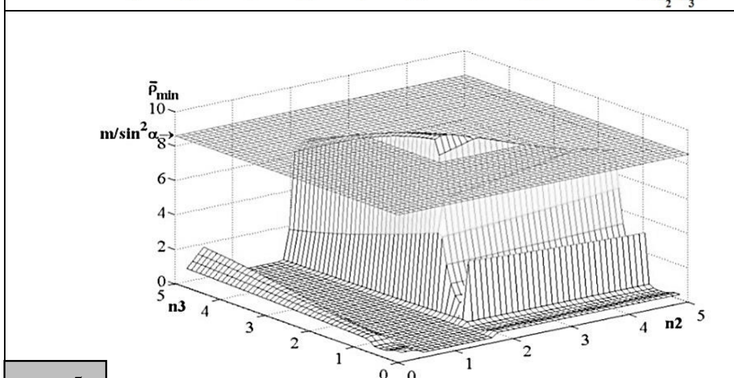
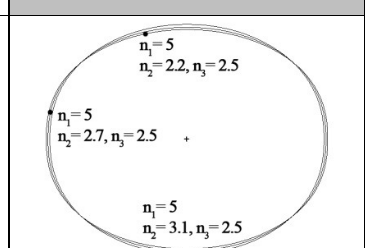
	 $n = 3$, $n_1 = 5$, $n_2 = n_3 \in [2, 3.3]$
 $n_1 = 5$	 $n = 3$, $n_1 = 5$, $n_2 \in [2,3.5]$, $n_3 = 3$

Table 4. *Variation of centrode minimal curvature radius and geometry for a = b*

	 $n = 4$, $n_1 = 7$, $n_2 = n_3 \in [2,2.8]$
 $n_1 = 5$	 $n = 4$, $n_1 = 5$, $n_2 \in [2,2.8]$, $n_3 = 2.5$

For supershapes with equal lengths of the major axes (tab. 3), it can be noticed that for equal exponents n_2 and n_3, the higher are their values, the larger is the range of the curvature radius variation that lead to potential gear pitch curve. For $n_2 = n_3 < 0.5$, the variation of $\overline{\rho_{min}}$ also indicates the possibility of generating the gear by rack-cutter, but the graphical representation of the supershape excludes it due to the pinched lobes [17]. While $n_2 \neq n_3$, the selection of the optimal parameter follows approximately the above mentioned pattern; the shape of the gear pitch curve slightly differs from the previous case in the sense of asymmetric scaling along the major axes, as different exponents influence the sin-cos terms.

4. Noncircular gear generation by rack-cutter

The generation of tooth involute profiles for planar noncircular gears, based on a standard rack cutter, requires a specific kinematics in order to provide the rolling motion. The basic idea [Litvin] is illustrated in Fig. 1a, where gear and rack centrodes are in mesh in T point, while the rack cutter is translated with velocity v_{ct}, along the common tangent (t) to centrodes, and the gear is both rotated about the centre of rotation O_1, with angular velocity ω_{rr}, and translated along the perpendicular to (t) direction, with velocity v_{rt}.

The simulation of the gear generation is based on the following set up geometric and kinematic initial data:

- the both gear blank and rack cutter are related to a fixed coordinated system, $O_fX_fY_f$, with Y_f axis along the common tangent to centrodes, and the origin O_f in the initial position of the contact point ,T_0 (Fig. 1b);
- the $O_1X_1Y_1$ rotating coordinate system is rigidly attached to the gear blank (its own polar system). While the gear and its system are rotated by γ angle, the origin O_1 also is translated along the fixed X_f axis, at distance x_{rt} from O_f (fig. 1c);
- the noncircular gear centrode is described by the equation of the supershape (Eq. 1') and scaled to a standard length of πmz, where m is the gear modulus and z – number of teeth;
- the rack cutter is defined by a standard geometry of m modulus and pressure angle $\alpha = 20°$;
- the motions of gear blank and rack cutter, defined by the corresponding γ gear rotational angle – x_{rt} gear translating distance – y_{ct} rack cutter translating distance, are expressed as:

$$x_{rt}(\theta_1) = -r(\theta_1) \cdot \sin \mu(\theta_1) \qquad (4)$$

$$y_{ct}(\theta_1) = -s(\theta_1) + r(\theta_1) \cdot \cos \mu(\theta_1) \qquad (5)$$

$$\gamma(\theta_1) = \theta_1 + \mu(\theta_1) - \pi/2 \qquad (6)$$

where $\mu\,(\theta_1)$ defines the orientation of the tangent (t) to the gear centrode, at current T point,

$$\mu(\theta_1) = arctg \frac{r(\theta_1)}{\frac{dr(\theta_1)}{d\theta_1}} \qquad (7)$$

and $s(\theta_1)$ is the distance of rolling, respectively the length of the arc T_0T:

$$s(\theta_1) = T_0T = TO_c = \int_{-\theta_1}^{0} \frac{r(\theta_1)}{\sin \mu(\theta_1)} d\theta_1 \qquad (8)$$

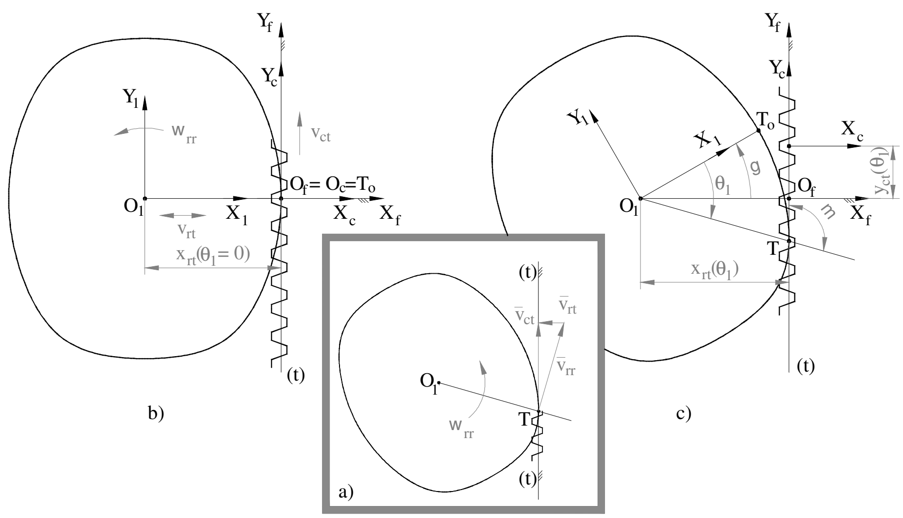
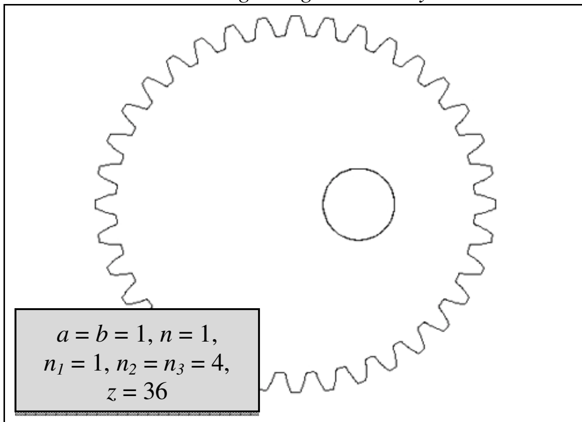
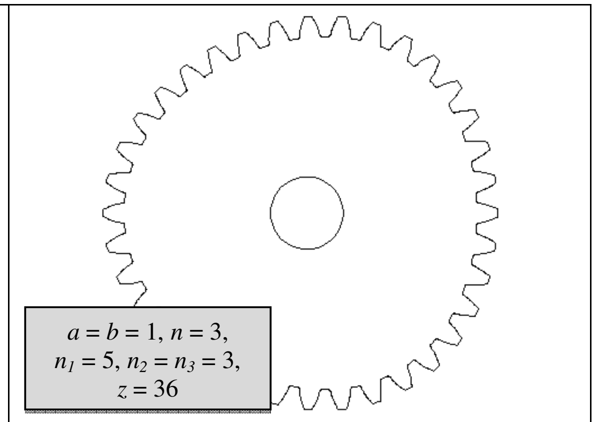
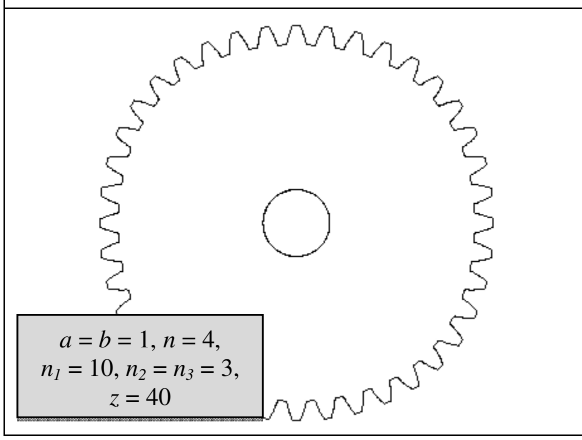


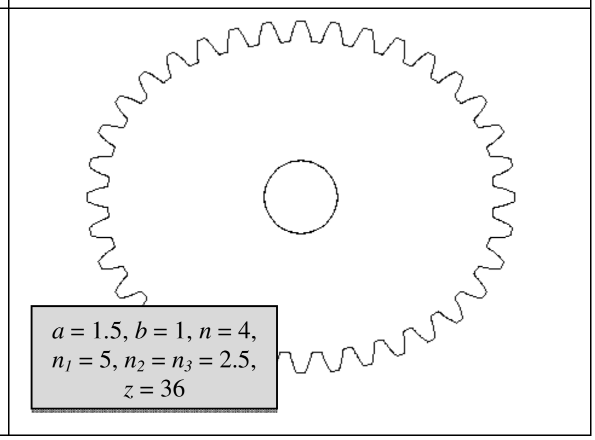
Fig. 1. *Illustrating rolling geometry and kinematics*
a) related motions of gear blank and rack cutter; b) initial position of centrodes; c) current position of centrodes

Table 5. *Noncircular gears generated by rack cutter*









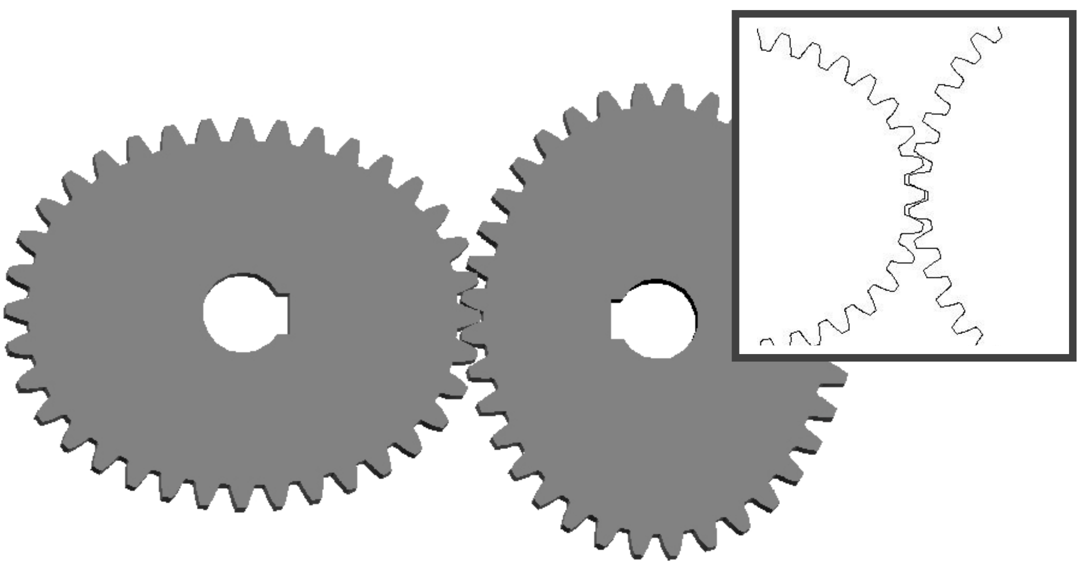


Fig. 2. *Illustrating a pair of conjugated noncircular gears, generated by rack cutter ($a = 1.5$, $b = 1$, $n = 4$, $n_1 = 5$, $n_2 = n_3 = 2.5$, $z_1 = z_2 = 36$)*

The animation of the noncircular gear generation is developed in AutoCAD environment, based on the interference of Matlab and AutLISP codes, as the rolling algorithm requires numerical approaches of the above mentioned functions (eq. 4-8). The precision of the gear generation is obviously influenced by the precision in both computer applications; the algorithm is performed with a precision of 10^{-15} in Matlab and transferred to the AutoCAD precision of 10^{-8}. Table 5 illustrates noncircular gears generated by a standard rack-cutter (m = 2 mm, α = 20°) rolling on the supershape of the gear centrode, designer by properly chosen defining parameters, as indicated. Figure 2 presents a pair of conjugated noncircular gears, generated by the same rack cutter.

5. Conclusions

As noncircular gears keep challenging the designers in gear industry, the authors initiated a new approach for the gear pitch curve generation. In the attempt of generalizing the non-circular gear design, as the first step in the design procedure is the centrode modelling, the Gielis' superformula is proposed to generate symmetrical or asymmetrical, closed or open, lobed circular or elliptical pitch curves. The choice of the supershape six defining parameters was the priority in optimal selection of the supershape in order to develop a further suitable method for the gear teeth generation. The paper is focused on non-circular gears, with convex pitch curves, generated by a standard rack cutter, based on an adequate kinematics. Specific codes were designed to perform the gear manufacturing simulation in AutoCAD environment, connected to Matlab resources. Studies on noncircular gear generation, considering complex concav-convex pitch curves, will further be considered.

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REFERENCES

[1]. **Litvin, F., Fuentes, A.,** *Gear Geometry and Applied Theory*, Second edition, 2004, Cambridge University Press.

[2]. **Chang, S.-L., Tsay, C.-B.**, *Computerized tooth profile generation and undercut analysis of noncircular gears manufactured with shaper cutters*, "Journal of mechanical design", Vol. 120, 1998, pp. 92-99.

[3]. **Danieli, G. A.,** *Analytical description of meshing of constant pressure angle teeth profiles on a variable radius gear and its applications*, "Journal of mechanical design", Vol. 122, 2000, pp. 123-129.

[4]. **Bair, B.-W.**, *Computer aided design of elliptical gears*, "Journal of mechanical design", Vol. 124, 2002, pp. 787-793.

[5]. **Figliolini, G., Angeles, J.,** *The synthesis of elliptical gears generated by shaper-cutters*, "Journal of Mechanical Design", Vol. 125, 2003, pp. 793-801.

[6]. **Tsay, M.-F., Fong, Z.-H.,** *Study on the generalized mathematical model of noncircular gears*, "Mathematical and computer modelling", Vol. 41, 2005, pp. 555-569.

[7]. **Mundo, D.,** *Geometric design of a planetary gear train with non-circular gears*, "Mechanism and machine theory", Vol 41, 2006, pp. 456-472.

[8]. **Litvin, F. L., et al.,** *Generation of planar and helical elliptical gears by application of rack-cutter, hob and shaper*, "Computer methods in applied mechanics and engineering", Vol. 196, 2007, pp. 4321-4336.

[9]. **Riaza, H. F. Q., et al.,** *The synthesis of an N-lobe noncircular gear using Bezier and B-Spline nonparametric curves in the design of its displacement law*, "Journal of mechanical design", Vol. 129, 2007, pp. 981-985.

[10]. **Li, J.-G., et al.,** *Numerical computing method of noncircular gear tooth profiles generated by shaper cutters*, "Int. J. Manuf. Technol", Vol. 33, 2007, 1098-1105.

[11]. **Xia, J., et al.,** *Noncircular bevel gear transmission with intersecting axes*, "Journal of mechanical design", Vol. 130, 2008, pp. 054502-1-7.

[12]. **Jing, L.,** *A pressure angle function method for describing tooth profiles of planar gears*, "Journal of mechanical design", Vol 131, 2009, pp. 051005-1-8.

[13]. **Yang, J., et al.,** *On the generation of analytical noncircular multilobe internal pitch curves*, "Journal of mechanical design", Vol 130, 2008, pp. 092601-1-8.

[14]. **Gielis, J., Beirinckx, B, Bastiaens, E**., *Superquadrics with rational and irrational symmetry*, Symposium on Solid Modeling and Applications, 2003.

[15]. **Gielis, J.**, *A generic geometric transformation that unifies a wide range of natural and abstract shapes*, "American Journal of Botany", 90, 2003, pp. 333–338.

[16]. **Litvin, F. L., et al.,** *Design and investigation of gear drives with non-circular gears applied for speed variation and generation of functions*, "Computer methods in applied mechanics and engineering", Vol. 197, 2008, pp. 3783-3802.

[17]. **Andrei, L., Vasie, M.,** *Using supershape in noncircular centrode modeling process*, Annals of "Dunarea de Jos" University of Galati, Mathematics, Phisics, Theoretical Mechanics, Fascicle II, Year II (XXXIII), 2010, pp. 259-266.

[18]. **Litvin, F., et al.,** *Noncircular gears. Design and generation*, Cambridge University Press, 2009.