

## AN INVESTIGATION INTO THE PREDICTION OF OPTIMAL MACHINING CONDITIONS FOR POLYSTYRENE FOAM CUT WITH A TAUT HOT-WIRE

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### ABSTRACT

*A series of tests were undertaken in which the feed force, wire temperature and kerf width were recorded for expanded polystyrene foam (EPS) and extruded polystyrene foam (XPS) being cut with a hot-wire cutting-tool. From the gathered data general relationships between cutting parameters, material properties and kerf width (cut surface accuracy) were sought and developed.*

*The findings reported in this paper provide an insight into the associated hot-wire cutting mechanics and the potential for consistently applying optimal cutting conditions during automated machining operations.*

**KEYWORDS:** plastic foam, cutting, hot-wire, automation.

### 1. INTRODUCTION

Very little research information or commercial data is available on hot-wire cutting of plastic foam. A rigorous literature search has revealed nothing in the way of standard cutting procedures or process optimisation techniques. This lack of information has stimulated interest in this area by the authors, and a number of papers have been produced to-date as a result [1- 3].

The central aim of the current research is to achieve optimum performance (workpiece accuracy and surface texture) from the hot-wire cutting process. This paper presents experimental data and findings gained from rigorous cutting trials carried out by the authors, and essentially provides details on how the main cutting parameters influence the quality of the cut. The parameter interrelationships are explored and to conclude, a new predictive model for optimum automated cutting is proposed.

The specific elements of research reported herein are:

- Cutting force, feed-rate and wire temperature fundamental inter-relationships,
- Hot-wire temperature and kerf width analyses leading to cut surface accuracy predictions,
- Effective heat input and volume/mass specific effective heat input,
- The development of a predictive mathematical model for optimum automated foam cutting.

### 2. TEST APPARATUS

The cutting trial apparatus comprised of an industrial robot, hot-wire cutting device, load-cell, thermocouple, electrical power supply and foam sample holding fixture. These system components are introduced and discussed below:

- The workpiece manipulation device was a Kuka KR6/2, 6-axis articulated robot which was fitted with the foam sample holding fixture. The robot effected the accurate linear movement of the foam samples over the statically mounted hot-wire, at predefined velocities.
- The purpose built hot-wire cutting device supported a representative 140 mm length of Nikrothal N80 wire. The wire was tensioned using a pneumatically actuated (cylinder) tensioning device. Thermal expansion and contraction of the wire was accommodated by setting the cylinder pressure to a constant value which thus provided constant wire tension throughout the tests. The cutting device was statically mounted to the load cell (PT Ltd., Model PT1000 with maximum load capacity of 3kg) which in turn was mounted on a heavy steel frame to eliminate erroneous readings due to vibration and inertial forces.
- An Omega thermocouple ( $\varnothing$  0.1" sheathed K-type, part number KQIN-18U-12) was fixed onto the hot-wire where it was used to monitor the local wire temperature. As the electrical

potential, applied across the hot-wire, adversely affected the thermocouple readings, the driving power supply (detailed below) was momentarily switched off while temperature readings were being taken. For a given unmodulated current setting the modulated current was controlled to ensure that an equivalent power was delivered.

- Electrical heating of the wire was achieved by way of a programmable regulated power supply (ITECH, Model IT6831). The unit adopted for the tests could deliver a maximum current of 10A at 18V.

Figure 1 below shows a close up view of the purpose built cutting device, foam workpiece, thermocouple and wire tensioning device (far right).

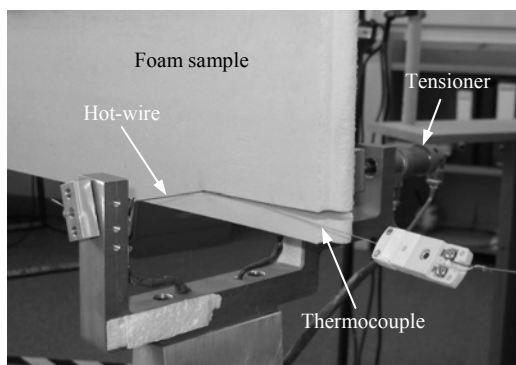


Fig.1. Test apparatus with thermocouple measuring active hot-wire cutting temperature

### 3. TEST PROCEDURES

Three different types of plastic foam were employed in the cutting trials. They were 15kg/m<sup>3</sup> Expanded Polystyrene (S-Grade Polyfoam by Bondor), 26kg/m<sup>3</sup> Expanded Polystyrene (H-Grade Polyfoam by Bondor) and 30kg/m<sup>3</sup> Extruded Polystyrene (Styrofoam by DOW).

The cutting trials consisted of over 400 individual cuts from which a significant number of measurements were made. The general cutting procedure for all the cutting tests was as follows:

- A sheet of material was held vertically in the clamping device which was manipulated by the robot.
- Power was supplied to the hot-wire by fixing the average current and allowing the voltage to float. The temperature of the wire was then allowed to stabilise.
- A robot programme was run to initiate the linear cutting of the foam sample at a predetermined feed-rate.
- As a result of step c above, a 10mm sliver of material was pared from the parent sheet by the hot-wire. The cutting force was monitored throughout and logged to a computer, and the surface finish was subsequently examined.

- The robot programme was then necessarily modified to advance the cutter path by 10mm. Steps b-d were then repeated measuring the temperature in the centre of the sample throughout the cut. This was done independently of the force measurement to ensure the thermocouple did not introduce errors into the force measurement.
- Steps b to e were then repeated changing the test conditions (current/wire temp and feed rate) as necessary.

The above procedure was used to produce the bulk of the data that underpins this paper, including the cutting force, electrical current, feed rate and surface texture relationships. However an additional series of tests was carried out to obtain measures for the cut sample kerf widths (approximately 70 cutting tests were conducted).

## 4. RESULTS

### 4.1. Characteristic cutting force

Throughout the cutting trials the cutting force for each sample was measured and logged with the cutting time. The resulting cutting force versus time graphs provide insight into the cutting mechanisms and provide proof of the transient nature of hot-wire cutting. A characteristic ‘‘S’’ shape force profile was found that is consistent for all foam samples of sufficient length. Figure 2 shows the cutting force profile of a 30mm wide XPS sample cut with a feed rate of 1100mm/min and a free air wire temperature of 500°C (electric current of 6A). The low cutting force associated with stage I is caused by a high initial wire temperature, which melts the advancing foam before it is able to contact the wire. In stage III the wire is in thermal equilibrium with the surrounding air/foam and depending on its temperature may exhibit a wide range of cutting conditions from vaporisation to ripping.

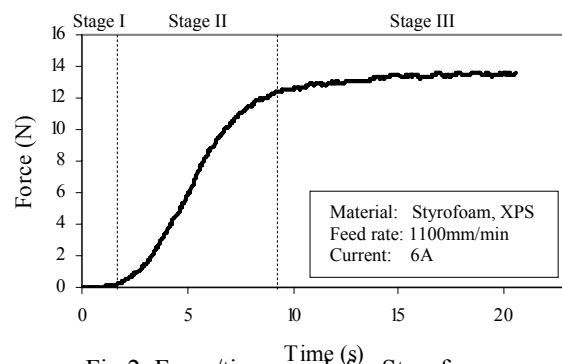


Fig.2. Force/time graph for Styrofoam showing 3 distinctive cutting stages

Analysing the various cutting force graphs revealed the anti-symmetric properties of feed rate and electrical power. Increasing the feed rate has the

same effect on the cutting force as reducing the electrical power (current) and vice versa.

**4.2. Wire temperature dynamics**

With the recorded hot-wire temperature superimposed on the associated cutting force results the dynamic behaviour of the interplaying hot-wire and work-piece becomes evident (see Fig.3 below).

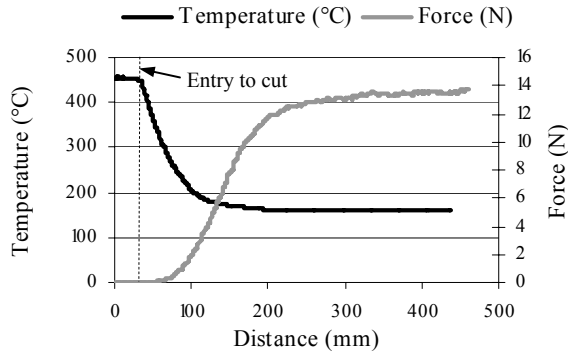


Fig.3. Wire temperature/feed-force relationship for extruded polystyrene

The hot-wire does not initially contact the foam and only does so when the heat field surrounding the wire ‘collapses’. This finding implies that the cut surface of the foam will be further away from the wire at the start of the cut than at the end of the cut, where mechanical cutting forces are typically present. Casual observations of the surface form of hot-wire cut samples confirm that the wire temperature does indeed have an effect on both the longitudinal and transverse profiles. Figure 4 illustrates typical longitudinal and transverse profiles and highlights characteristic features which are present, to varying degrees, in all samples cut without hot-wire temperature control.

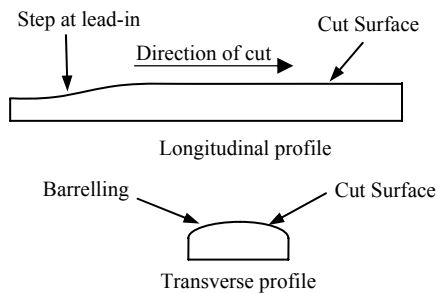


Fig.4. Longitudinal and transverse profiles of typical hot-wire cut samples

An earlier investigation [4] into the transverse temperature profile revealed the degree to which foam on the edge of the sample experienced higher temperatures. For the analysed case, the very edge of the foam experienced wire temperatures 60°C hotter than in the middle of the cut. This gradient had a

visible effect on the surface form of the cut sample, as per Fig. 4.

The thermal gradient in the wire at the edge of the sample causes the barrelling effect on the foam cut surface. This barrelling effect is amplified with higher power inputs and higher feed rates due to the greater temperature difference between the wire in free air and the wire in cut.

The authors have found that whilst transverse temperature gradients for straight wires are unavoidable, they can be minimised by using lower wire temperatures and slower feed rates. The barrelling can also be kept constant over time by modulating the hot-wire electric current: Maintaining a constant temperature profile throughout a cut means the wire will be in thermal equilibrium and thus a desirable surface may be produced for the entire length of the cut [5]. The reported adaptive temperature control method allows the *step at lead-in* to be virtually eliminated.

**4.3. Kerf width**

The finite diameter of a hot-wire necessarily generates a gap during cutting, through which the hot-wire passes. This gap is known as the kerf, the width of which is determined by the temperature and feed rate of the wire. Large kerf widths can introduce uncertainty into the dimensional accuracy of the cut (see section 4.2) and should be minimised. The kerf width can be reduced by using smaller wires, however the kerf is often much wider than the wire itself due to the thermo-mechanical nature of hot-wire cutting. Consequently it is important to understand the relationship between electrical current (wire temperature) and kerf width.

The kerf width test results shown in Fig.5. reveal that the initial kerf width is considerably wider than the final kerf width along the length of a cut (for a constant wire current).

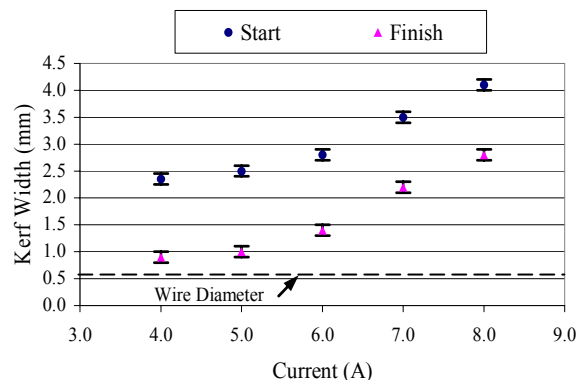


Fig.5. Kerfwidth vs. current for XPS cut at 0.0150m/s

High energy cuts have kerf widths much wider than the diameter of the hot-wire; more than six times the wire diameter in the worst case of Fig. 5. For low currents the kerf width approached the theoretical minimum of the hot-wire diameter (0.64 mm). The

error bars, in the graph, represent the repeatability of the robot and the measurement errors.

If the wire temperature can be successfully controlled in realistic cutting situations then the kerf width will be constant over the length of the cut and a simple temperature dependant offset may be applied to the cutting paths to negate the effects of kerf error and wire diameter/radius. It has been demonstrated [5] that accurate surfaces can be generated to within  $\pm 0.05$  mm.

#### 4.4. Area specific effective heat input

Heat is created within a hot-wire cutting tool by passing an electric current through the electrically resistive metal wire. This is known as the Joule heating effect and is defined by the following equation.

$$Q = I^2 \cdot R \quad (1)$$

Where the units of  $Q$ ,  $I$  and  $R$  are power (Watt), current (Ampere) and resistance (Ohm) respectively. In the case of a hot-wire the heat generated can be defined as a linear volumetric heat flux as shown in (2).

$$Q_L = \frac{Q}{L_e} \quad (2)$$

Where  $L_e$  represents the length of wire in the cut. In order to consider the influence of the heat input and the cutting speed together, the effective heat input is described, as shown in (3).

$$Q_{eff} = \frac{Q_L}{v_c} \quad (3)$$

Where  $v_c$  is the cutting velocity in m/s and  $Q_{eff}$  has the units  $W \cdot s/m^2$  or  $J/m^2$ . Ahn et al from the Korea Advanced Institute of Science and Technology (KAIST) have carried out a number of experiments investigating the relationship between the effective heat input, cutting speed and kerf width [6 - 8].

Within this report the effective heat input will be renamed the area specific effective heat input, to open the way for subsequent developments. However, the renamed area specific effective heat input will continue to be represented by the same term,  $Q_{eff}$ .

Physically this value represents the amount of electrical/thermal energy used to create a unit area of cut surface. It is a very useful parameter as it allows a wide range of cutting data to be presented on a single graph. Ahn et al used  $Q_{eff}$  to observe how the kerf width changes with energy input, however much more information can be gained if the cutting force is also plotted on a secondary axis [4]. Figure 6 shows the relationship between cutting force per unit length of wire and kerf width against the area specific effective heat input for H grade EPS.

The force data follows an exponential decay or ‘elbow’ trend across the range of  $Q_{eff}$ . This ‘elbow’ represents a change from one dominant cutting mode to another; the high forces on the left represent

thermo-mechanical cutting while the almost nonexistent forces on the right represent pure thermal cutting. The kink in the elbow represents smooth and generally preferred cutting conditions. The kerf width varies linearly over the experimental range of parameters despite the fact the range of cutting conditions included both high force, low power thermo-mechanical cutting, as well as low force, high power pure thermal cutting.

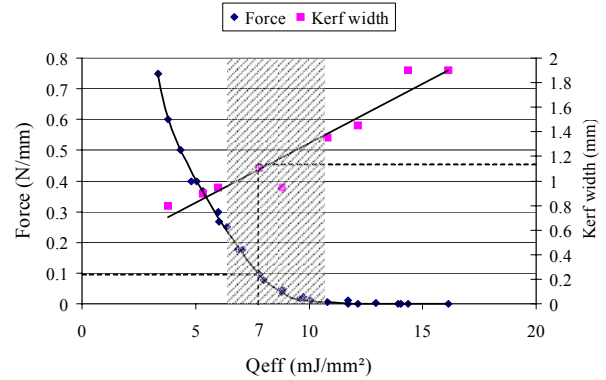


Fig.6. Force and kerf width vs. effective heat input for H Grade EPS

It is not hard to see how the cutting data displayed in the above graph can be used to predict the surface texture and kerf width of future cuts (assuming the same material and wire diameter are used). For example, to obtain a smooth surface in VH-grade EPS a small non-zero cutting force is required, this relates to an effective heat input of approximately  $7 \text{ MJ/mm}^2$  (as shown by the dotted line). This also correlates to a kerf width of  $1.1 \text{ mm}$ , which can then be used to determine the wire offset needed to negate the effects of kerf on the dimensional accuracy of the surface. In this case the wire offset would be  $0.55 \text{ mm}$  (half of  $1.1 \text{ mm}$ ).

#### 4.5. Volume specific effective heat input

The concept of unifying the power and the feed rate to represent the amount of energy used to create a cut surface can be taken one step further [5]. Dividing the area specific effective heat input,  $Q_{eff}$ , by the corresponding kerf width produces the volume specific effective heat input (4).

$$^{vol}Q_{eff} = \frac{Q_{eff}}{\lambda} \quad (4)$$

Where:

$$Q_{eff} = \text{area specific effective heat input in } J/m^2$$

$$\lambda = \text{kerf width in m}$$

The volume specific effective heat input,  $^{vol}Q_{eff}$ , physically represents the amount of electrical/heat energy needed to ‘melt’ a unit volume of foam and has the dimensions  $J/m^3$ . In this situation the term ‘melt’ is used to express the process by which the foam near the wire reduces in volume (not necessarily

actual melting). This is a function of the temperature dependant foam material properties, the cutting tool geometry and heat transfer characteristics therefore it would be extremely challenging to determine from first principles.

Figure 7 below shows the volume specific effective heat input for XPS with different wire diameters plotted against the area specific effective heat input. It is clearly evident that all of the cuts have the same  $^{Vol}Q_{eff}$  value (within the experimental errors) independent of the  $Q_{eff}$  and the diameter of the wire. The average  $^{Vol}Q_{eff}$  value over the entire XPS hot-wire cutting trials was found to be 10.1 MJ/m<sup>3</sup> with a standard deviation of 10%. The corresponding value for EPS is 7.71 MJ/m<sup>3</sup> with a standard deviation of 10%.

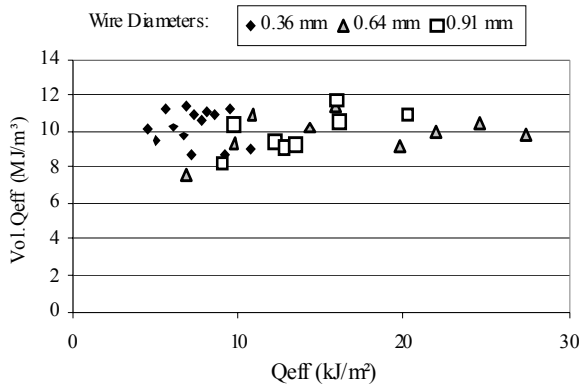


Fig.7. Volumetric effective heat input for XPS cuts with different wire diameters

The first noteworthy point to be drawn from fig. 7 is that the energy required to create a unit volume of kerf in XPS (or EPS) with a hot-wire is constant (within the experimental range of parameters). The second point to note is that this is true regardless of the wire size and material (i.e. it is independent of wire size and material). The third point is that once the  $^{Vol}Q_{eff}$  value is known for a given plastic foam material, it is possible to calculate the kerf width for any wire, feed rate and power input combination within a practical range of parameters.

To demonstrate the utility of this development, consider the following scenario:

*A CNC foam cutting operator is required to cut a block of XPS foam with high dimensional accuracy. Before the cut is made the operator must decide what electric current and feed rate to use and what the resulting kerf width will be. The operator can then set a wire offset to counter the effect of the kerf and achieve the desired block dimensions without trail and error. If, for example the foam cutting machine was fitted with a 0.36 mm diameter Nichrome wire (with a resistance of 10.58 Ω/m), and the operator chose a current of 3A and a feed rate of 0.0150 m/s, the kerf width can be calculated from a rearranged version of (4).*

The method follows:

$$^{Vol}Q_{eff} = \frac{Q_{eff}}{\lambda}$$

$$^{Vol}Q_{eff} = \frac{Q_L}{v_c} \cdot \frac{1}{\lambda}$$

$$^{Vol}Q_{eff} = \frac{I^2 \cdot R}{v_c} \cdot \frac{1}{\lambda}$$

Rearranging for  $\lambda$  gives

$$\lambda = \frac{I^2 \cdot R}{v_c} \cdot \frac{1}{^{Vol}Q_{eff}} \quad (5)$$

Substituting the known values provides the solution.

$$\lambda = 0.63\text{mm}$$

#### 4.6. Mass specific effective heat input

For plastic foams which have identical molecular structures and differ only by density, such as S, H and VH grade EPS, the volume specific effective heat input can be converted into an even more powerful tool by dividing  $^{Vol}Q_{eff}$  by the density of the material (6).

$$^{Mass}Q_{eff} = \frac{^{Vol}Q_{eff}}{\rho_f} \quad (6)$$

Where:

$^{Mass}Q_{eff}$  = the mass specific effective heat input

$\rho_f$  = density of the foam.

The mass specific effective heat input is the same for all EPS foams of different densities that have the same molecular structure [5]. With this development it is possible to carry out the same exercise for EPS as was previously done with XPS above, however this time the density of the material can also be a variable. The mass specific heat input provides the basis for a powerful predictive mathematical model relating all the important parameters in hot-wire foam cutting i.e. the kerf width, power, feed rate, material and cutting tool.

#### 4.7. Hot-tool cutting calculator

A hot-tool cutting calculator (HCC) was developed in Excel that allows any of the variables in (5) and (6) to be calculated if the other five variables are known. At the core of the HCC the  $^{Mass}Q_{eff}$  value of 297 kJ/kg for EPS was adopted: evaluated from experimental data collected using 26 kg/m<sup>3</sup> EPS. To validate the ability of the HCC to determine the kerf widths of foams with different densities, four calculations were made to predict the kerf width of cuts made with 15 kg/m<sup>3</sup> EPS. This is important because the 15 kg/m<sup>3</sup> kerf width measurements were not used to calculate the  $^{Mass}Q_{eff}$  and therefore provided an independent verification. The calculated

kerf widths were then compared with the experimentally found line of best fit. Figure 8 below shows that the calculated kerf widths match well with the experimentally determined line of best fit, however there is a constant small negative offset of approximately 0.15 mm. One possible reason for the slight negative offset is the fact that the 15 kg/m<sup>3</sup> EPS was obtained from a different supplier than the 26 kg/m<sup>3</sup> EPS and may have a slightly different composition. However, as there is a 10% standard deviation in the  $^{Mass}Q_{eff}$  values the experimental data is within the error range of the HCC predictions.

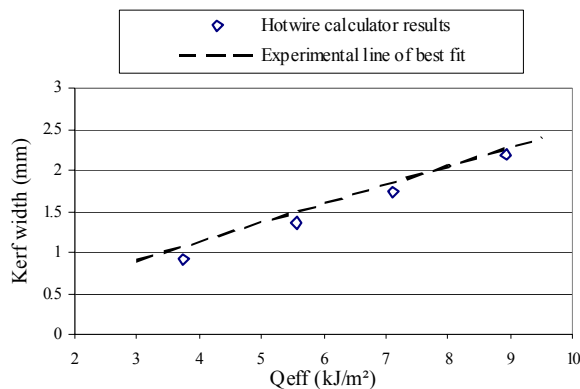


Fig.8. Hot-wire calculator results vs. the experimental line of best fit.

With further experimentation the HCC facility could be extended to handle a broad range of foam materials. In turn the database and controlling equations could be embedded into appropriate automated tool path generation software for hot-wire foam cutting/sculpting.

In robotically actuated foam cutting/sculpting applications the robot controller is not always able to maintain a constant cutting speed throughout a complete cutting sequence. This is due to rotational velocity and acceleration limits, which may prevent the tool from reaching its ideal speed. Using the equations embedded in the HCC with the proposed user defined tool and material libraries, automated tool path generation software could reduce cutting time by simultaneously varying the feed rate and power to maintain constant kerf widths and surface texture. The outline steps detailed below indicate how this could be implemented.

1. Roughing/finishing tool paths automatically generated in CAM software from part digital definition (solid model).
2. Hot-wire cutting tool power requirements determined through manipulation of: Tool definition library, Material definition library and Mathematical model of section 4.4 – 4.7 above.
3. Tool power and tool path control routines are synchronised.
4. Data transformed into a form recognised by the robot and hot-tool controllers.

5. Control program loaded onto robot PC, foam blank mounted/referenced, program is executed

## 5. CONCLUSIONS

Not being able to control the kerf width could be a serious source of error when sculpting 3D objects. In the test conducted, the kerf width was found to vary between 1.4 to 6 times the diameter of the hot-wire.

New parameters were developed called the volume specific effective heat input and the mass specific effective heat input. Once obtained, these terms can be used to predict the kerf width that will result from using any foam/wire combination.

A hot-tool cutting calculator (HCC) was developed utilising the mass effective heat input to provide an effortless way to predict the cutting conditions for a previously untested range of cutting conditions. To the authors knowledge the mass specific heat input and the mathematical model behind the HCC is a new development in the understanding of foam cutting mechanics and provides a significant step toward the goal of automated and intelligent sculpting technology.

A method of incorporating the equations embedded in the HCC into CAM software was suggested which could greatly increase the speed, accuracy and utility of automated sculpting technology.

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