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THE CORRECTIVE PROFILING MODELING OF RACK TOOL FOR INVOLUTE TEETHED GEAR GENERATION

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ABSTRACT

The modality to generate surfaces by enwrapping using rolling methods with tools as rack-gear, gear shaped and hub worm is very used at the involute teethed wheels machining, as so as, for the ordered curling whirl: splined shaft, teeth of the milling tools, pump worm for helical compressor etc.
As in all the manufacturing process the machined surfaces are affected by errors. In this paper is proposed a model for the generating error compensation at the machining with rack-gear, regarding the generation principle and the constructive form of the rack-gear tool, when the surface to be generated is known only approximately at discrete points. Bezier polynomials are used to elaborate a specific methodology for profiling tools. The results we have obtained suggest that the tool profile errors are small enough to be used in engineering applications.
Based on original software, numerical examples regarding the rack-gear tool's corrective profiling are presented.

KEYWORDS: cutting, modelling, research area etc.

1. INTODUCTION

The problem of machining precision increasing was developed by the synthesis of numerical models for prediction and compensation of the generating errors, Lee [1], using on machine measurement systems, Cho [2], or by the determination of the optimal position of cutting tool, Li [3] and regarding the blank and regarding the composite forms, Fulin [4].

The correction of the tool's profile that generates through the ordered whirl of profiles: rack-tool, gear shaped tool and rotary cutter, so it would compensate for some generating errors are, in general, difficult to resolve, due of the large number of tool's cutting edges (number of teeth), which take part in generating process, Litvin [5], Radzevich [7].

Imagining some methodologies that allow the correction of shape to all of the cutting edges for all these types of tools, so it would compensate the errors in the generated surface, usually an orderly whirl of surfaces (profiles) can be a useful way of improving the performance of generation, through this procedure, the modeling at the mathematical level of this kind of generating process, Cucu [6].

It's proposed in the following work a method for correction of the rack tool for generating an involute tooth gear is applied.

2. THE INVERSE GENERATION PROCESS

The problem of generating the involute tooth-gear implies a process starting from knowing the rack-tool that as a result of the movements in juxtaposition to the centrode associated to the blank product, generates the surface of the gear flank, see figure 1.

The reference systems are presented the same significations as in figure 1.

It implies that the profile of the rack tool in a numerical form is known, through array S,

$$S = \left\| \xi_i \quad \eta_i \right\|^T, i = 1, 2, ..., n, \tag{1}$$

in the system ξηζ with „n" being a number large enough to describe the shape of the transversal profile of the rack tool. At the same time, the relative movement is defined according to the reference system of the winding of the surfaces to be generated, joined with the system XYZ,

$$X = \omega_3(\theta)[\xi + a]; \; a = (-R_{rp} \quad -R_{rp}\theta \quad 0)^T, \qquad (2)$$

where the array ξ_i, η_i has the signification given in (1).

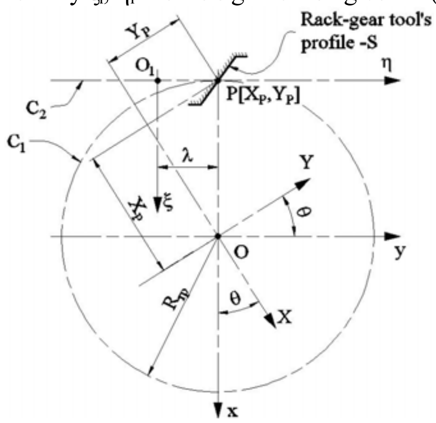

Fig. 1. The inverse generation

It is determined by the family of profiles, in the reference system XY:

$$(S)_\theta \left| \begin{array}{l} X = \left[\xi_i - R_{rp}\right]\cos\theta + \left[\eta_i - R_{rp}\theta\right]\sin\theta; \\ Y = -\left[\xi_i - R_{rp}\right]\sin\theta + \left[\eta_i - R_{rp}\theta\right]\cos\theta, \end{array} \right. \qquad (3)$$

with ξi, ηi, the coordinates of the current point of the array and θ – the variable angular parameter.

The winding of the family of profiles (3) represent the generated surface through the winding of the rack-tool of profile (1).

The condition of winding, through the method of the minimal distance is [8]:

$$d = \left(\sqrt{\left(X_{(\xi_i\eta_i)} - X_p\right)^2 + \left(Y_{(\xi_i\eta_i)} - Y_p\right)^2}\right)_{\min} \qquad (4)$$

where Xp, and Yp, are the coordinates of the gearing pole.

$$X_p = -R_{rp}\cos(\theta); \; Y_p = -R_{rp}\sin(\theta), \qquad (5)$$

And $X(\xi_i, \eta_i), Y(\xi_i, \eta_i)$ are the given coordinates by the family (3).

The limits of variation of the parameter θ are:

$$\theta_{\min} = -\frac{m}{R_{rp}}\left[\frac{1}{tg\alpha} + tg\alpha\right], \theta_{\max} = \frac{1,2m}{R_{rp}}\left[\frac{1}{tg\alpha} + tg\alpha\right], \qquad (6)$$

with m - modulus of the generating rack tool ;

α – angle of gear on the base circle, of the radius R_{rp}.

The ensemble of equations (3) and (4) represent, in a discrete form, the generated profile of the rack tool for which the transversal profile is given in (1).

3. THE ALGORITHM FOR CORRECTIVE MODELING OF RACK TOOL FOR INVOLUTE PROFILE

It is accepted that through measuring the generated surface, named in the fallowing the real generated surface (real generated profile), it is determined, in discrete form, a profile of the generatrix of a cylindrical surface pertaining to a whirl of surfaces (profiles) associated to a centroide, see fig. 3, in this way:

$$G_E = \left\| \begin{array}{cccc} X_{E_1} & X_{E_2} & \dots & X_{E_n} \\ Y_{E_1} & Y_{E_2} & \dots & Y_{E_n} \end{array} \right\|^T \qquad (7)$$

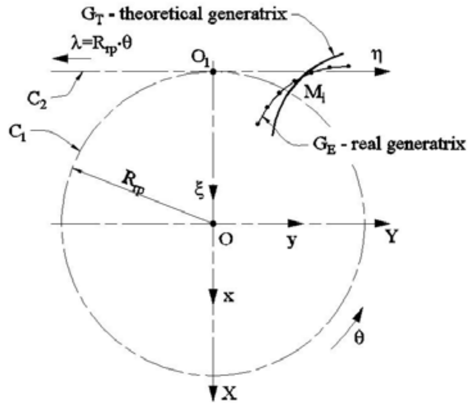

Fig. 2. Real and theoretically generatrix

It is determined that this real side generatrix G_R is not identical with the theoretical (side) generatrix G_T as a target of generating. After processing the two curves, profiles G_T and G_R, the real sources can not be established which cause G_T and G_R to not coincide. For the case in which the fabrication restarts in the technological conditions an algorithm is proposed through which the selection of the new target of the process in the form of a new generatrix called: virtual generatrix, following the process, the real generatrix will be closer to (possibly overlaying) the theoretical generatrix.

The model of the virtual generatrix G_F is proposed to be the mirror image of the curve G_R juxtaposed to the theoretical generatrix. This is done with the idea that we have disturbing factors in the technological process acting in the same direction this will lead to obtaining a new generatrix G_R closer to the shape and dimensions of the theoretical generatrix.

The model of the virtual generatrix is accepted to be fig. 3,

$$M_{F_i} \left| \begin{array}{l} X_{F_i} = X_{E_i} + (1+\delta)\, d_{\min} \sin\alpha_i; \\ Y_{F_i} = Y_{E_i} + (1+\delta)\, d_{\min} \cos\alpha_i; \quad i = 1,2,\dots n, \end{array} \right. \qquad (8)$$

for the current point of the virtual generatrix.

It is understood by $d_{\min}$ – the minimum distance between the current point of the real generatrix and that of the theoretical generatrix [8]

$$d_{\min} = \sqrt{\left(X_{Ei} - X_{Ti}\right)^2 + \left(Y_{Ei} - Y_{Ti}\right)^2} \qquad (9)$$

and $0 < \delta \le 1$

$$tg\alpha_1 = \left|\left(Y_{Ti+1} - Y_{Ti}\right) / \left(X_{Ti+1} - X_{Ti}\right)\right|. \qquad (10)$$

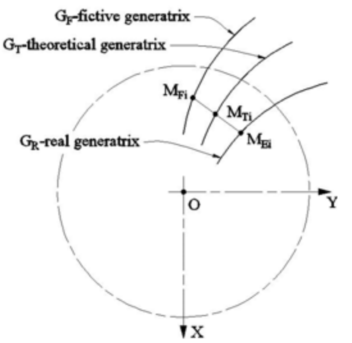

Fig. 3. Virtual generatrix

Therefore a target of generation is obtained, the profile G_F as an array of coordinates:

$$G_F = \begin{Vmatrix} X_{F_1} & X_{F_2} & \dots & X_{F_n} \\ Y_{F_1} & Y_{F_2} & \dots & Y_{F_n} \end{Vmatrix}^T \qquad (11)$$

based of which can be used to make the new cylindrical target surface of generation with the rack-tool.

The generation of the new target surface (profiles) implies the realization of a new shape of the rack-tool that must be the reciprocal winding to the new profile.

3.1. The algorithm for rack tool profiling

If now it is considered that the new target (profile) is described by the array found in equation (13) in the relative movement (12). This new profile generates a family of profiles known in a discrete form such as is found in equation (11):

$$\xi = \left[\omega_3^T(\theta) \begin{Vmatrix} X_{F_i} \\ Y_{F_i} \end{Vmatrix} - a \right]; \quad i = 1,2,...,n. \qquad (12)$$

Or, after the development:

$$(G_F)_\theta \begin{vmatrix} \xi_{F_i} = X_{F_i}\cos\theta - Y_{F_i}\sin\theta + R_{rp}; \\ \eta_{F_i} = X_{F_i}\sin\theta + Y_{F_i}\cos\theta + R_{rp}\theta; \end{vmatrix} \qquad (13)$$

$i = 1,2,...,n,$

The condition of winding associated to the family of trajectories (G_F) θ for the method of the minimal distance [8] see equation (14) is:

$$d = \left(\sqrt{\left(\xi_{(\xi_i\eta_i)}\right)^2 + \left(\eta_{(\xi_i\eta_i)}\right)^2} \right)_{\min} \qquad (14)$$

with $i = 1,2,...,n.$

The ensemble of the point of the massive that satisfies condition (14) for different sizes of the parameter θ represents the virtual profile of the rack tool, reciprocal winding for the new target surface, be it SF the new form of the rack tool profile.

$$S_F = \begin{Vmatrix} \xi_i \\ \eta_i \end{Vmatrix}^T, \quad i = 1,2,\dots,n. \qquad (15)$$

The virtual profile, S_F, of the rack-tool generator represents the winding virtual profile G_F of the blank.

This solution of the correction problem becomes unpractical taking into consideration the complexity of this tool.

Therefore, an approximate solution of necessary correction of the rack-tool is proposed, so it can generate the target surface - virtual generatrix G_F.

$$A_\alpha \begin{vmatrix} \xi = \xi_i - \lambda\sin\alpha; \\ \eta = \eta_i; \\ \zeta = \lambda\cos\alpha. \end{vmatrix} \qquad (16)$$

The real geometric cutting edge is given through intersecting the primary peripheral surface (cylindrical surface (16)) with the rack surface (in the first phase it was considered the plane surface)

$$P_\gamma \begin{vmatrix} \xi = \xi_v - t\cos\gamma; \\ \eta = t\sin\gamma, \end{vmatrix} \qquad (17)$$

with t – variable parameter, γ – rack angle; ξv – the coordinates of the point on the tools profile found on the axis ξ, at the shortest distance of the revolving axis of the profile to be generated.

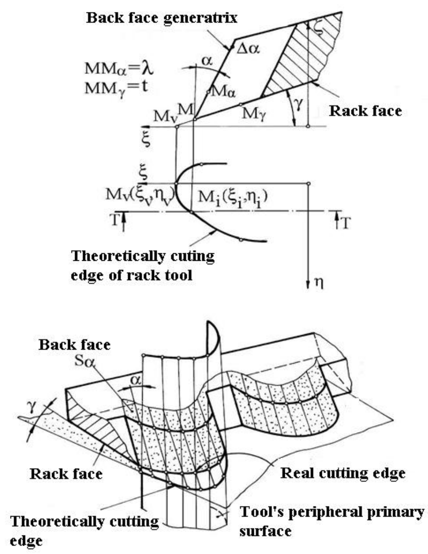

Fig. 4. Rack gear's active surfaces

The real cutting edge of the rack tool belonging simultaneously to the rake and primary peripheral surfaces as given by the equations:

$$\begin{cases} \xi_e = \xi_i - [\xi_i - \xi_v] \dfrac{\sin\gamma}{\cos(\alpha-\gamma)}\sin\alpha; \\ \eta_e = \eta_i; \\ \zeta_e = [\xi_i - \xi_v] \dfrac{\sin\gamma}{\cos(\alpha-\gamma)}\cos\alpha. \end{cases} \qquad (18)$$

It is obviously that only for γ=0 the real cutting edge is identical to the theoretical profile of the rack-tool.

A way of correcting the rack-tool profile can be imagined by, the modification of the geometry of the active part of the rack tool (the size of the angles α and γ) so that the real profile of the rack-tool becomes closer in shape and dimension to the corrected profile of the rack-tool, as was defined previously. We call this profile –

approximated profile. It is sure, that only in particular cases those two profiles, approximated and corrected, can coincide but can be similar enough. Correcting in this way is relatively easy to do without concern the functionality of the rack tool and its capacity to be brought back to the initial form of the profile through re-sharpening, taking into consideration the initial size of the geometric parameters (α and γ).

4. APPLICATION

The proposed methodology is applied for the determination of the approximate form of the rack tool generated of the involute teeth belonging to a gear with straight teeth having the following characteristics: z = 42 teeth; m=5 mm, x=0 addendum modification.

The profile (in a transversal section of the gear axis to be manufactured) measured on a 3D measuring machine which is representing in table 1.

Table 1. The real and virtual piece profile coordinates

X_E [mm]	Y_E [mm]	X_F [mm]	Y_F [mm]
-100.031	2.347	-99.998	2.462
-100.352	2.414	-100.334	2.528
⋮	⋮	⋮	⋮
-109.844	6.297	-109.686	6.254
-107.555	5.125	-109.811	6.341

The virtual profile is determined (new target) see (12) and (13) in juxtaposition with the theoretical profile see table 2. Starting from the virtual profile, the new target of the generation process, table 2, the corrected profile of the rack tool is calculated and is represented the coordinates of profile to be generated.

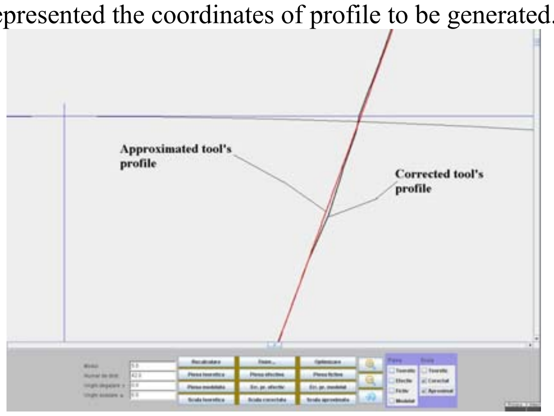

Fig. 5. Java applet-scaled view (rack gear profiles)

Table. 2. The approximated and corrected tools

ξ_A [mm]	η_A [mm]	ξ_C [mm]	η_C [mm]
5.568	1.900	5.614	1.903
5.539	1.911	5.599	1.908
⋮	⋮	⋮	⋮
-4.209	5.459	-4.219	5.494
-4.238	5.470	-4.207	5.491

The corrected profile is compared with the approximated one. In this way is proof that the errors between these not exceed an imposed size.

For the numeric results of the problem a software program was made using the java programming language, the results being presented in the form of an applet having a dialog window that permits the introduction of input data, the modification of the size of the geometric parameters (α and γ), the assessment of the size of the error of the approximated profile juxtaposed to the theoretical profile, the graphic layout of the real and virtual form of the approximated tooth and the corrected profile of the rack tool.

5. CONCLUSIONS

It has been proven that we can have a corrected form of the rack-tool generator which, starting from the predicted error of generating – the virtual profile to be generated – can be substituted with a form of rack-tool having a profile obtained through the modification of geometric parameters (α and γ) – the approximated profile.

Obviously, is possible the successive corrections of the rack-tool, regarding the error of the profile (the form of the real generated profile) for different manufacturing of the toothed gears.

The example illustrated the accuracy of the method. Besides the computational advantage (faster execution), the main appeal of the method is that the profile to be generated can be represented by the coordinates of a small number of points. These points can be eventually obtained from physical measurements on the generatrix.

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