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END MILL AND PLANNING TOOL'S PROFILING FOR GENERATION OF DISCREETLY KNOWN HELICAL SURFACES

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ABSTRACT

The helical surfaces, especially those with identically anti-homologous flanks, may be generated with end mill tools. The constructive advantages and the using domain of these tools are known.

In this paper, are presented an algorithm, based on the Bezier approximation polynomials for the tool's profiling designated to generate helical surfaces. The algorithm regarding a reduced number of points on the in-plane generatrix of the helical surface to be generated (3 or 4 points) may constitute a simple and fast variant for this tool's profiling.

Is analysed the cylindrical tool's profiling —planning tools— for helical surface in the same approach manner.

Are presented numerical examples, based on a software dedicated for this problem and realized in the java programming language, which proof the proposed method quality, versus a theoretically method.

KEYWORDS: and mill tool, planning tool, helical surfaces.

1. INTRODUCTION

There are multiple solutions when the helical surfaces, especially those which form flutes with identical anti-homologous flanks (the involute flanks of the teeth of helical gear, flanks of the modulus thread, thread for ball circulating screw) are generated by milling with end mill tool [2], [4], [6].

The constructive simplicity of the tool [4] which generate, in the cutting movement, a peripheral revolving surface, generating simultaneous the both gauge flank, the small number of the tool (often only two teeth) make the machining and use of these tool's type to be more advantageous opposite to a disk tool. Obviously, the productivity of machining using this tool (end mill) is more reduced, and, as follow, this solution should be economically used only for a small number of the same type pieces [3], [4].

The profiling of this tool's type often follows the same steps as for disk-tool's profiling, the particularities of the end mill tool imposing its profiling algorithm.

Exist situations when, for helical surfaces machined as unique pieces, or for repairs, when the end mill realization is too expensive, and as follow is imposed to choose a more simple method, the milling being replaced with the planning of helical surface flank, generating a cylindrical surface, reciprocally enwrapping with the helical surface, generating with planer tool of the helical surface.

In the both situations, the surface to be generated may be known by a small point number, often as results of the measuring of a generatrix of this (not necessary an in-plane generatrix) allowing a discrete expression for the helical surface to be generated.

In this paper, is proposed a methodology for end mill tool's profiling and planer tool for the case of the discrete expression with a small points number (3 or 4 points) of its generatrix profile.

One of the goals is to compare the numerical results obtained with the proposed algorithm with the results obtained with profiling theoretically methods [2], [6], for the same surfaces types, in order to proof the new method quality.

2. END MILL TOOL'S PROFILING

The in-plane generatrix of the helical surface, here regarded in *XY* plane of the reference system, is substitute by a Bezier polynomials, with 2 or 3 degree, regarding the point's number considered (measured) along the generatrix (3 or 4 points), [1].

They are defined the axis and the surface's helical parameter, for which will be determined the form of a revolving surface reciprocally enveloping with $\vec{V}(Z)$ axis and p parameter, as so as the point's coordinates:

$$A\left[X_{A,}Z_A\right];\ B\left[X_{B,}Z_B\right];\ C\left[X_{C,}Z_C\right] \tag{1}$$

on the generatrix known in discrete form, see Fig. 1.

If we mark with:

$$G\left|\begin{aligned}&X(\lambda)=\lambda^2 A_X+2\lambda(1-\lambda)C_X+\\&+(1-\lambda)^2 B_X;\\&Z(\lambda)=\lambda^2 A_Z+2\lambda(1-\lambda)C_Z+\\&+(1-\lambda)^2 B_Z,\end{aligned}\right. \tag{2}$$

the Bezier substitutive polynomials [1], then, the helical surface with $\vec{V}$ axis and p helical parameter may have an expression, principled, in form:

$$\Pi(\lambda,\varphi)\left|\begin{aligned}&X(\lambda,\varphi)=X(\lambda)\cdot\cos(\varphi);\\&Y(\lambda,\varphi)=X(\lambda)\cdot\sin(\varphi);\\&Z(\lambda,\varphi)=Z(\lambda)+p\cdot\varphi.\end{aligned}\right. \tag{3}$$

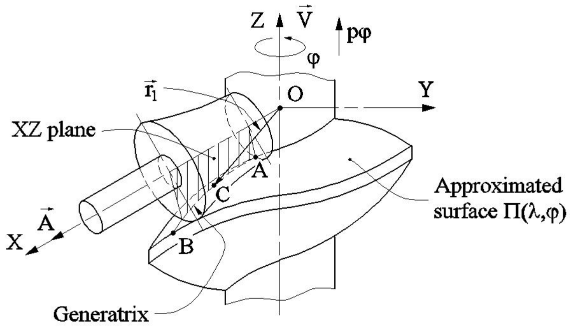

Fig. 1. End mill tool, reference system and tool's axis, $\vec{A}$

Note:

- The $X(\lambda)$ and $Z(\lambda)$ polynomials will be identified, regarding the known coordinates on G generatrix;
- Is possible to know also a spatial generatrix (a spatial curve), by simultaneous considering of its projections on the reference system's planes.

Starting from the (3) forms of the helical surface, $\Pi(\lambda,\varphi)$, may be defined the parameters of the normal at the substitutive surface, in forms:

$$N_\Pi:\left|\begin{aligned}&N_X=\dot{Y}_\lambda\cdot\dot{Z}_\varphi-\dot{Y}_\varphi\cdot\dot{Z}_\lambda;\\&N_Y=-\left[\dot{X}_\lambda\cdot\dot{Z}_\varphi-\dot{X}_\varphi\cdot\dot{Z}_\lambda\right];\\&N_Z=\dot{X}_\lambda\cdot\dot{Y}_\varphi-\dot{X}_\varphi\cdot\dot{Y}_\lambda.\end{aligned}\right. \tag{4}$$

The partial derivatives are calculated from (3) form of the helical surface.

For the tool's axis definition

$$\vec{A}=\vec{i} \tag{5}$$

and for the position vector of the current point on the $\Pi(\lambda,\varphi)$, surface,

$$\vec{r}=X(\lambda,\varphi)\vec{i}+Y(\lambda,\varphi)\vec{j}+Z(\lambda,\varphi)\vec{k} \tag{6}$$

the enwrapping condition [5], [6],

$$\left|\overrightarrow{N_\Pi},\vec{A},\vec{r}\right|\le q \tag{7}$$

with q—positive and very small (for example $q=1\cdot10^{-2}$), may be bring in form

$$\begin{aligned}&\left|\left[Z(\lambda)+p\cdot\varphi\right]\cdot N_Y-\right.\\&\left.-\left[X(\lambda)\cdot\sin(\varphi)\right]\cdot N_Z\right|\le q.\end{aligned} \tag{8}$$

Is determined the axial section of end mill cutter, see fig. 2:

$$S_A\left|\begin{aligned}&H=X(\lambda)\cdot\cos(\varphi);\\&R=\sqrt{Y^2(\lambda)\cdot\sin^2(\varphi)+\left[Z(\lambda)+p\cdot\varphi\right]^2}.\end{aligned}\right. \tag{9}$$

for λ and φ couples of values which meets the condition (7).

In all the presented algorithm stages, the profiles calculus is made only for 3 or 4, considered points belong to the profile.

For the axial section (9), known in discrete form, for 3 or 4 points on this, its made an approximation by a second (third) degree Bezier polynomials, determining a representation form for this.

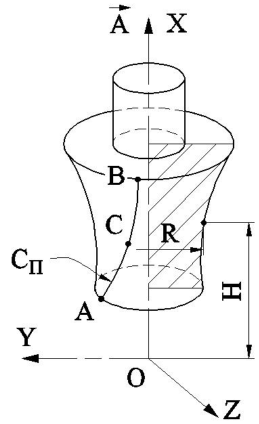

Fig. 2. Axial section of end mill cutter

2.1. End mill tool for generating a worm with circular axial section

This surfaces type appears in construction of ball circulating screw or screws for traction with increased fatigue resistance.

They are presumed known (measured) the coordinates of three points on the helical surface generatrix profile, see Fig. 3 and table 1.

The center coordinates of circle arc which represent the discreetly known generatrix of the helical surface are

$$O_C\left(X_{O_C},0,Z_{O_C}\right). \tag{10}$$

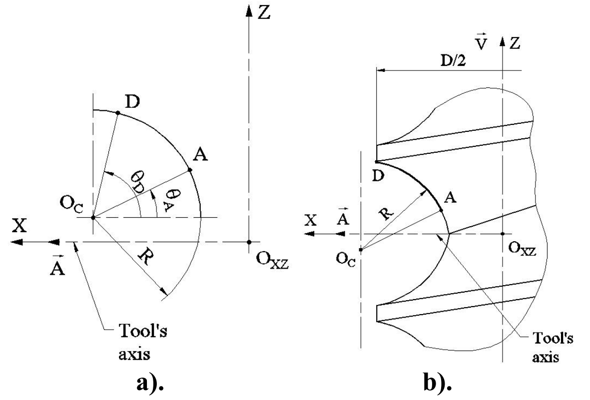

Fig. 3. a). Generating profile — circle arc in axial plane; b). Ball circulating screw

The identification algorithm for the Bezier polynomial substitutive of the helical surface generatrix profile is presented in table 1.

Table 1. Coefficients for a 3rd degree polynomial

θ	Generatrix primary profile	λ	Polynomials coefficients
θ_A	$X_A = X_{O_C} - R\cos\theta_A$ $Z_A = Z_{O_C} + R\sin\theta_A$	0	$D_X = X_A$ $D_Z = Z_A$
θ_B	$\theta_B = \theta_A + \dfrac{\theta_D - \theta_A}{3}$ $X_B = X_{O_C} - R\cos\theta_B$ $Z_B = Z_{O_C} + R\sin\theta_B$	$\dfrac{1}{3}$	$B_X = \dfrac{18\cdot X_C - 9\cdot X_B}{6} +$ $+\dfrac{2\cdot X_A - 5\cdot X_D}{6}$ $B_Z = \dfrac{18\cdot Z_C - 9\cdot Z_B}{6} +$ $+\dfrac{2\cdot Z_A - 5\cdot Z_D}{6}$
θ_C	$\theta_C = \theta_A + \dfrac{2(\theta_D - \theta_A)}{3}$ $X_C = X_{O_C} - R\cos\theta_C$ $Z_C = Z_{O_C} + R\sin\theta_C$	$\dfrac{2}{3}$	$C_X = \dfrac{-5\cdot X_A + 2\cdot X_D}{6} +$ $+\dfrac{18\cdot X_B - 9\cdot X_C}{6}$ $C_Z = \dfrac{-5\cdot Z_A + 2\cdot Z_D}{6} +$ $+\dfrac{18\cdot Z_B - 9\cdot Z_C}{6}$
θ_D	$X_D = X_{O_C} - R\cos\theta_D$ $Z_D = Z_{O_C} + R\sin\theta_D$	1	$A_Y = X_D$ $A_Z = Z_D$

As so was previously show, the p parameter value is known ($p = 3.18$).

In Fig. 4 and table 2, are presented the axial profile of end mill tool in a presentation approximated by 3rd Bezier polynomials regarding the same profile determined by an analytical method. Also, is presented the profiling error value regarding the profile determined by an absolutely rigorous analytical method, for:

$O_C = [52,0,0]$; R=8 mm;

D=60 mm; p=3.18 mm;

$\theta_A = 0$; $\theta_D = 0.87266$ rad.

Table 2.

λ	Approximated profile		Theoretical profile		Error [mm]
	H [mm]	R [mm]	H [mm]	R [mm]	
0.000	**44.000**	**0.080**	**44.000**	**0.080**	**0.000**
	44.011	0.422	44.010	0.423	0.002
	⋮	⋮	⋮	⋮	⋮
	44.286	2.109	44.286	2.111	0.002
0.333	**44.349**	**2.326**	**44.350**	**2.329**	**0.002**
	44.384	2.438	44.385	2.440	0.002
	⋮	⋮	⋮	⋮	⋮
	45.260	4.290	45.259	4.288	0.002
0.666	**45.319**	**4.382**	**45.318**	**4.381**	**0.002**
	45.451	4.575	45.449	4.573	0.002
	⋮	⋮	⋮	⋮	⋮
	46.579	5.859	46.577	5.859	0.002
1.000	**46.836**	**6.085**	**46.836**	**6.085**	**0.000**

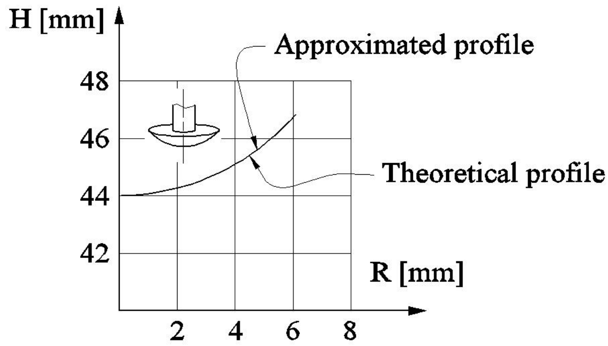

Fig. 4. Axial section of the mill end tool

The algorithm is completed with specialized software in java programming language. A specifically applet is presented in fig. 5.

The dialog boxes allow the defining of:

- O_C center coordinates of axial profile;
- R radius value [mm];
- θ_A and θ_D angles values;
- external surface diameter, D [mm];
- p helical parameter value [mm].

The applet allow to draw the axial section of end mill tool *(H, R)*; determination of profiles coordinates and the method profiling error level against a theoretically method [Nikolaev] [5].

In the applet a helical surface is presented, the end mill tool's axis and the normal at discreetly helical surface.

Note: When the points are measured on the helical surface's generatrix, be A; B; C; D these points, we defined the λ parameter values by:

$$\lambda_B = \frac{\left|\overline{AB}\right|}{\left|\overline{AB}\right| + \left|\overline{BC}\right| + \left|\overline{CD}\right|}; \lambda_C = \frac{\left|\overline{AB}\right| + \left|\overline{BC}\right|}{\left|\overline{AB}\right| + \left|\overline{BC}\right| + \left|\overline{CD}\right|}, \quad (11)$$

where $\left|\overline{AB}\right|, \left|\overline{BC}\right|, \left|\overline{CD}\right|$ are the *AB, BC* and *CD* straight line segment modulus.

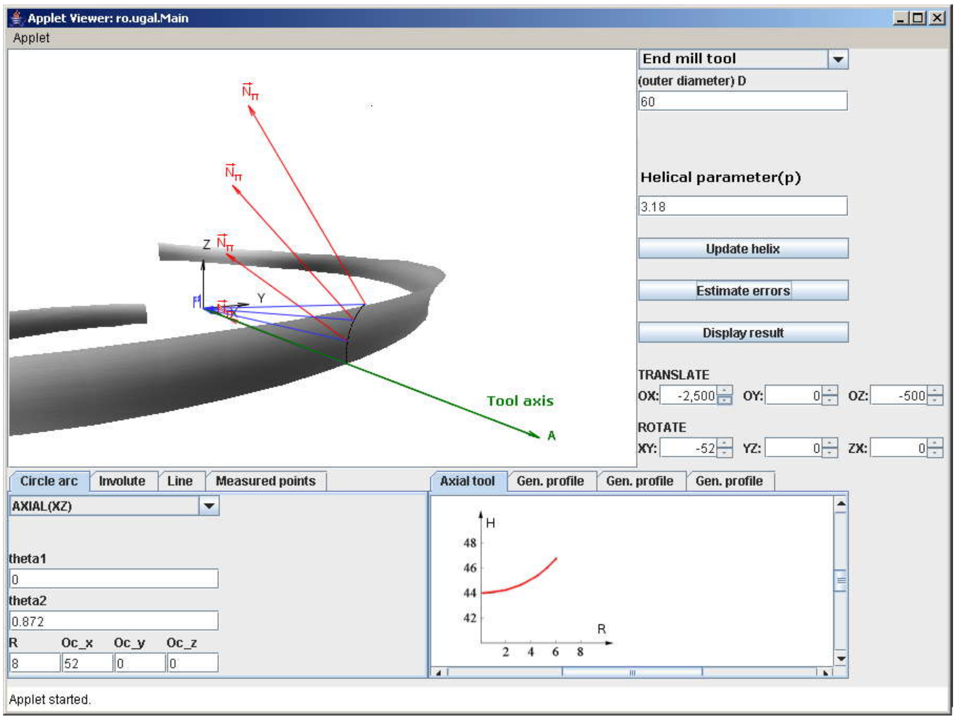

Fig. 5. Applet for end mill tool’s profiling

3. CYLINDRICAL TOOL’S PROFILING (PLANNING TOOL)

The cylindrical tool, (the planning tool) which generate in cutting movement a cylindrical surface, reciprocally enveloping with a helical surface known in discrete form, may be profiled based on an algorithm similarly with those previously presented.

For an in-plane generatrix, defined in discrete form and approximated by a Bezier polynomial with small degree, as result of knowing a small points number on the axial generatrix of the helical surface (3 or 4 points), is accepted as expression form of helical surface presented in discrete form, see the form (3).

The characteristically curve, C_{Π}, on the Π helical surface expressed in discrete form, as representing the curve tangent at the cylindrical surface (in Fig. 6, surface with generatrix perpendicularly on P_T plane and with C_{Π} as directrix) is defined based on a specifically enwrapping condition [5], [6]:

$$\vec{N}_{\Pi} \cdot \vec{t} = 0. \tag{12}$$

The P_T plane, crossing plane of the cylindrical surface, is a plane with contain the X axis and admit as normal the $\vec{t}$, the unitary vector of tangent at the helix with external diameter of helical surface Π.

In form (12), the Nikolaev condition [5] specifically for this enwrapping problem type was marked:

$\vec{N}_{\Pi}$ — normal at the helical surface expressed in discrete form;

$\vec{t}$ — the unitary vector of the cylindrical surface,

$$\vec{t} = \cos\alpha \cdot \vec{j} + \sin\alpha \cdot \vec{k}, \tag{13}$$

$$\alpha = \arctan\left({p}/{D_e} \right) \tag{14}$$

where:

- p is the helical parameter of surface (known);
- D_e — external diameter of the helical surface.

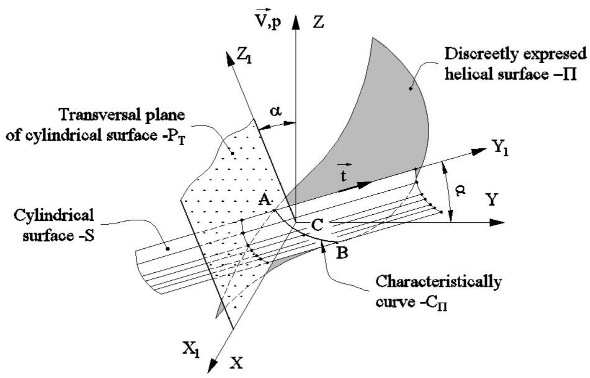

Fig. 6. Cylindrical surface and characteristically curve

In this way, the (3) and (12) equations assembly represent the characteristically curve of the two surfaces: the helical surface, expressed in discrete form and the cylindrical surface, as peripheral primary surface of the planning tool for helical surface generation.

In principle, the C_{Π} characteristically curve is expressed by its point’s coordinates,

$$C_{\Pi} = \begin{Vmatrix} X_{C_{\Pi},\lambda=0} & Y_{C_{\Pi},\lambda=0} & Z_{C_{\Pi},\lambda=0} \\ X_{C_{\Pi},\lambda=\frac{1}{2}} & Y_{C_{\Pi},\lambda=\frac{1}{2}} & Z_{C_{\Pi},\lambda=\frac{1}{2}} \\ X_{C_{\Pi},\lambda=1} & Y_{C_{\Pi},\lambda=1} & Z_{C_{\Pi},\lambda=1} \end{Vmatrix}, \quad (15)$$

for the helical surface approximation with a 2nd degree polynomial.

3.1. The S cylindrical surface

Being known the cylindrical surface generatrix direction, $\vec{t}$ and the characteristically curve form, C_{Π}, expressed in discrete form, is defined the S surface, in form:

$$\vec{R} = \vec{r} + k \cdot \vec{t}\ , \quad (16)$$

where

$$\vec{r} = X^{C_{\Pi}} \cdot \vec{i} + Y^{C_{\Pi}} \cdot \vec{j} + Z^{C_{\Pi}} \cdot \vec{k}\ , \quad (17)$$

vector of discrete points on the C_{Π} characteristically curve, k variable parameter.

Result, in principle, the S surface coordinates,

$$S: \begin{vmatrix} X^S = X^{C_{\Pi}}; \\ Y^S = Y^{C_{\Pi}} + k\cos\alpha; \\ Z^S = Z^{C_{\Pi}} + k\sin\alpha, \end{vmatrix} \quad (18)$$

which, by coordinate transforming

$$X_1 = \omega_1(\alpha) \cdot X\ , \quad (19)$$

lead to:

$$S_{X_1Y_1Z_1} \begin{vmatrix} X_1 = X^S; \\ Y_1 = Y^S\cos\alpha + Z^S\sin\alpha; \\ Z_1 = -Y^S\sin\alpha + Z^S\cos\alpha, \end{vmatrix} \quad (20)$$

representing the discrete cylindrical surface S in $X_1Y_1Z_1$ reference system, see figure 7.

The $X_1Y_1Z_1$ reference system is the reference system where the crossing plane of the cylindrical surface, P_T, is overlapped with X_1Z_1 plane.

The crossing section of discrete cylindrical surface (20) is obtained from condition

$$|Y_1| = q_1\ , \quad (21)$$

with q_1 arbitrary, positive and small, in form:

$$S_{P_T} \begin{vmatrix} X_1 = X^S; \\ Z_1 = -Y^S\sin\alpha + Z^S\cos\alpha, \end{vmatrix} \quad (22)$$

for k variable, see fig. 7.

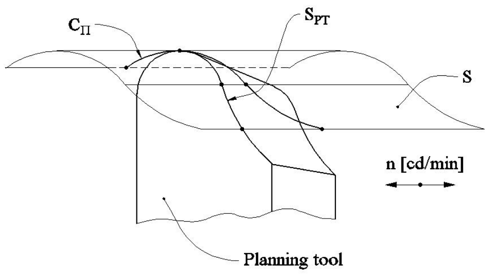

Fig. 7. Characteristically curve

3.2. Cylindrical tool for generation of helical surface with circular profile in axial plane

The helical surface is expressed in discrete form by an in-plane generatrix known by its points, fig. 8.

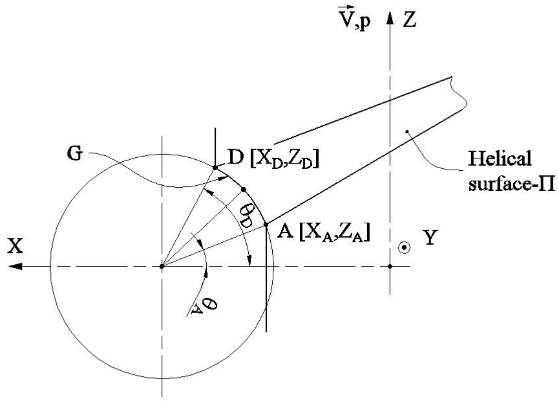

Fig. 8. Helical surface with circular generatrix in axial plane

Are presumed know the coordinates of helical surfaces generatrix.

Is identified the Bezier polynomial for G generatrix approximation, see table 2, for a 2nd degree polynomial or similarly for a superior degree polynomial.

On the helical surface:

$$\Pi \begin{vmatrix} X(\lambda,\varphi) = P_X(\lambda)\cos\varphi; \\ Y(\lambda,\varphi) = P_X(\lambda)\sin\varphi; \\ Z(\lambda,\varphi) = P_Z(\lambda), \end{vmatrix} \quad (23)$$

with $P_Y(\lambda)$, $P_Z(\lambda)$ substituting polynomials for discrete generatrix;

φ — variable angular parameter, is determined the φ and λ values which satisfied the (12) condition,

$$|N_Y\cos\alpha + N_Z\sin\alpha| \le q \quad (24)$$

q positive and small enough.

In this way is determined the characteristically curve on the S surface, helical surface discreetly expressed, and, from here, by (19), coordinates transforming, the crossing section form of the cylindrical surface S_{PT}.

In Fig. 9 and table 3, are presented the form and coordinates of the crossing section of cylindrical surface reciprocally enveloping with a worm with dimensional characteristics:

- coordinates of points belong to axial section, in mm – for a 3rd degree polynomial, see figure 3.a:

$$A\begin{vmatrix} X_A = 44.01 \\ Y_A = 0.39 \end{vmatrix}; B\begin{vmatrix} X_A = 44.36 \\ Y_A = 2.38 \end{vmatrix};$$

$$C\begin{vmatrix} X_C = 45.46 \\ Y_C = 4.61 \end{vmatrix}; D\begin{vmatrix} X_D = 46.85 \\ Y_D = 6.12 \end{vmatrix};$$

- helical parameter, p=3.18 mm.

It was elaborated a specialized software in java programming language, for this case, as applet, similarly with those presented in Fig. 5, where is determined:

- the crossing section of cylindrical surface, X,Z;
- the planning tool's profile coordinates $[X_1,Z_1]$;
- the profiling error level against a fundamental analytical method (Nikolaev).

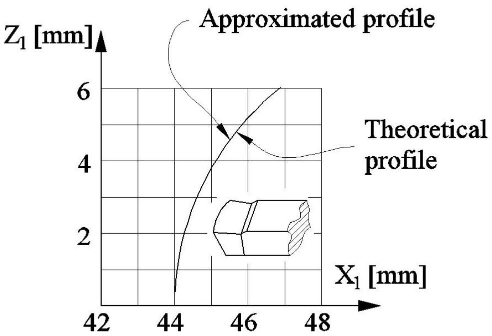

Fig. 9. Cylindrical tool's profile

Table 3.

λ	Approximated profile		Theoretical profile		Error [mm]
	X_1 [mm]	Z_1 [mm]	X_1 [mm]	Z_1 [mm]	
0.000	**44.010**	**0.395**	**44.010**	**0.396**	**0.000**
	44.033	0.720	44.032	0.720	0.001
	⋮	⋮	⋮	⋮	⋮
	44.353	2.312	44.352	2.311	0.000
0.333	**44.420**	**2.516**	**44.420**	**2.515**	**0.000**
	44.456	2.620	44.456	2.620	0.000
	⋮	⋮	⋮	⋮	⋮
	45.341	4.354	45.341	4.354	0.000
0.666	**45.400**	**4.439**	**45.400**	**4.439**	**0.000**
	45.530	4.619	45.529	4.619	0.001
	⋮	⋮	⋮	⋮	⋮
	46.635	5.808	46.634	5.807	0.001
1.000	**46.886**	**6.015**	**46.886**	**6.014**	**0.001**

The numerical results are presented against the results obtained using a fundamental method for planning tool's profile calculus.

The maximum error is 0.002 mm obtained for λ=0.125;

Is obviously that the tool's profile precision is exactly enough, and, for certain surfaces types, the poles representation method for in-plane generatrix of surfaces may be an alternative to the analytical method for helical surface generating tool's profiling.

The proposed method is characterized by the fact that the point number is relatively small, 3 or 4 points, the determination precision increasing may be obtained using substituting polynomials with superior degree.

The method has the advantage that allows the approach to the helical surface generation tool's profiling, starting from the known of some points measured on these surface.

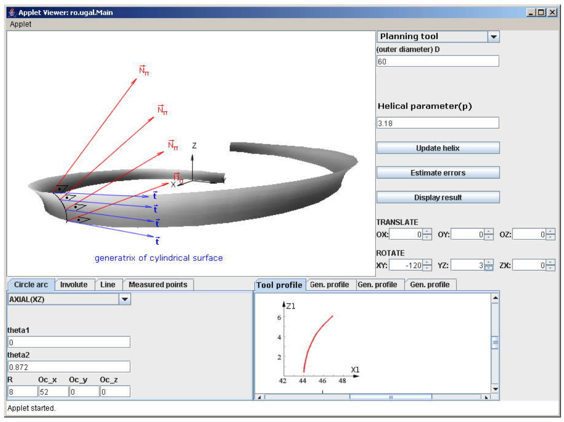

Fig. 10. Applet for cylindrical tool's profiling

4. SOFTWARE IMPLEMENATION

A software application was developed in Java programming language implementing the algorithm above. In Figs. 5 and 10 are presented specifically screenshots including representations of analytically and numerical generatrix for end mill and planning tools. Numerical results can be exported to comma separated values files.

5. CONCLUSIONS

The helical surfaces generating tools profiling method (end mill tool and cylindrical tool) is characterized by:

- the method is fundament on the reciprocally enveloping surfaces theory;
- the algorithm is applicable for helical surfaces known even for a small point number (3 or 4 points) which defined straight line segments or curve arcs;
- the Bezier polynomials coefficients for discreetly known generatrix are presented in tables;
- the method may be used also in case of points known by measuring;
- the tool's profiling precision, by the proposed method, is equivalent with results obtained using an analytical method;
- the method is fast and easy to apply.

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REFERENCES

1. **Favrolles, J. P.**, *Les surfaces complexes,* Hermes, 1998, ISBN 2-86601-675.a;
2. **Litvin, E. L.**, *Theory of Gearing*, Reference Publication 1212, NASA, Scientific and Technical Information Division, Washington D.C., 1984;
3. **Radzevich, S. P.**, *Kinematic Geometry of Surface Machining*, CRC Press, London, ISBN 978-1-4200-6340-0, 2008;
4. **Shaw, M.**, *Metal Cutting Principles*, Claredon Press Oxford, 1984;
5. **Lukshin, V. S.**, *Theory of Screw Surfaces in Cutting Tool Design*, Machinostroyenie, Moscow, 1968;
6. **Oancea, N.**, *Surfaces Engineering Through Winding*, "Dunărea de Jos" University Foundation Publishing House, Galați, 2004, ISBN 973-627-106-4;