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The Design of Wires and Fibers Extrusion System

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Abstract: *This paper analyses the wires/fibers extrusion process and develops a study of the cooling channel optimum length dependence on the material and process parameters: the Peclet number and the cooling channel propertiesc. As a consequence, guidelines are presented for wires and fibers extrusion system design.*

Keywords: *extrusion, design, temperature, industrial installation, natural convection.*

1. Introduction

The wires and fibers extrusion is a research domain that received great attention in the last decades [1÷3]. The complete knowledge of the phenomena that are taking place after the wires/fibers extrusion in the cooling tunnel can help us not only to understand the influence of different process or material parameters on the process development but also to design the technological process and installation.

This research subject is of special interest not only for the wires/fibers extrusion but for hot rolling, glass fibers drawing, continuous casting, etc. The general problem is concerning a cylinder with a high initial temperature coming out from the die or furnace and moving horizontally (or vertically, in some cases) with a constant velocity. An aiding or opposing fluid flow is generating a forced convection process that accelerates the cooling process.

This paper is making a step forward in the analysis of wires/fibers extrusion. It considers a horizontal cylindrical wire/fiber moving with a constant velocity and a cooling fluid moving in the same direction. Steady-state situation was considered and the finite differences method was used to solve the governing equations in dimensionless form.

After the velocity and temperature fields calculation, the definition of optimum design for the cooling system, Lopt, was realised. The Lopt dependence on different physical and process parameters was analyzed in an intense calculation process. Consequently, the parameters that are influencing Lopt in a significant manner were established and a design formula for the industrial installation was established for three extruded materials: teflon, glass and aluminum.

2. Mathematical model

Figure 1 presents the cylindrical workpiece of radius r_s, the workpiece that emerges from the extrusion die or furnace. It moves with the constant velocity u_s while the cooling fluid moving in the same direction determines a forced convection process with the initial velocity u_∞. The fluid has an initial temperature T_0. The cooling tunnel length is L_t. Neglecting the gravitational force influence, the problem becomes symmetric and only the upper half will be analyzed in a cylindrical coordinates system.

The fluid governing equations for mass, momentum and energy as well as the workpiece governing equation for the energy are considered in vorticity (ω) — stream function (ψ) formulation:

$$u = \frac{1}{r}\frac{\partial \psi}{\partial r};\ v = -\frac{1}{r}\frac{\partial \psi}{\partial x};\ \omega = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial r}, \qquad (1)$$

where: (u, v) is the fluid velocity field with its two components on the x and r axis.

The governing equations and the whole problem are transformed in dimensionless form (Fig. 1b) using the dimensionless variables [1] defined by equation (2), where t is time, α_s /α are the workpiece/fluid thermal diffusivities, ν is the kinematic viscosity, $Pe=u_s r_s/\alpha_s$, $Pr=\nu/\alpha$ and $Re= u_s r_s/\nu$ are Peclet, Prandtl and Reynolds numbers.

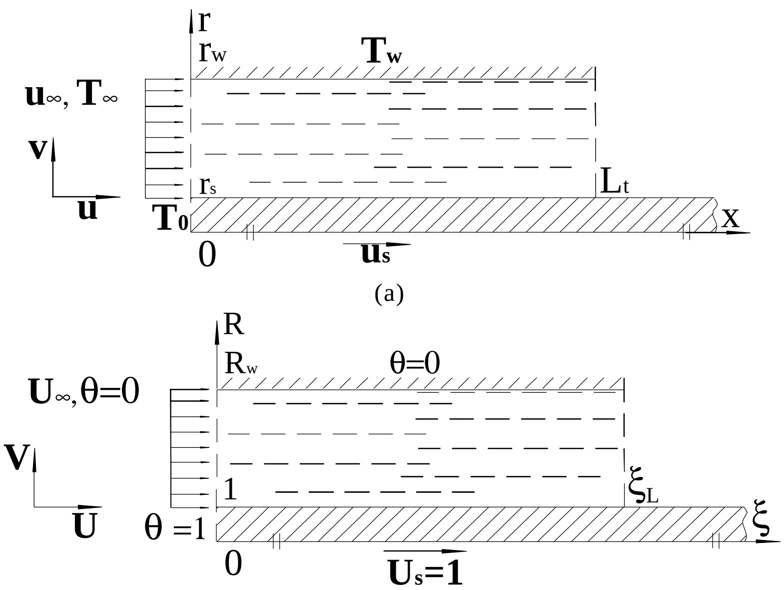


Fig. 1. Wires and fibers extrusion (a); dimensionless problem (b).

$$X=\frac{x}{r_s};R=\frac{r}{r_s};U=\frac{u}{u_s};V=\frac{v}{u_s};\Psi=\frac{\psi}{u_s r_s^2};$$

$$\Omega=\frac{\omega r_s}{u_s};\tau=\frac{tu_s}{r_s};\theta=\left(\frac{T-T_\infty}{T_0-T_\infty}\right);$$

$$Pe=\frac{u_s r_0}{\alpha_s};Pr=\frac{\nu}{\alpha};Re=\frac{u_s r_s}{\nu}, \qquad (2)$$

Having in view the difference between the x and r dimensions and a higher precision, the transformation $X=e^{\xi}-1$, defines the (ξ, R) computational domain and the governing equations:

$$\frac{1}{Re^{2\xi}}\frac{\partial^2\Psi}{\partial\xi^2}+\frac{\partial}{\partial R}\left(\frac{1}{R}\frac{\partial\Psi}{\partial R}\right)=-\Omega+\frac{\partial\Psi}{\partial\tau}; \qquad (3)$$

$$\frac{\partial\Omega}{\partial\tau}+\frac{1}{e^{\xi}}\frac{\partial(U\Omega)}{\partial\xi}+\frac{\partial(V\Omega)}{\partial R}=$$
$$\frac{1}{Re}\left[\frac{1}{e^{2\xi}}\frac{\partial^2\Omega}{\partial\xi^2}+\frac{\partial}{\partial R}\left(\frac{1}{R}\frac{\partial(R\Omega)}{\partial R}\right)\right]; \qquad (4)$$

$$\frac{1}{e^{\xi}}\frac{\partial(U\theta)}{\partial\xi}+\frac{1}{R}\frac{\partial(RV\theta)}{\partial R}=$$
$$=\frac{1}{Re\cdot Pr}\left[\frac{1}{e^{2\xi}}\frac{\partial^2\theta}{\partial\xi^2}+\frac{1}{R}\frac{\partial}{\partial R}\left(R\frac{\partial\theta}{\partial R}\right)\right]; \qquad (5)$$

$$\frac{1}{e^{\xi}}\frac{\partial\theta}{\partial\xi}=\frac{1}{R}\left[\frac{1}{e^{2\xi}}\frac{\partial^2\theta}{\partial\xi^2}+\frac{1}{R}\frac{\partial}{\partial R}\left(R\frac{\partial\theta}{\partial R}\right)\right]. \qquad (6)$$

The boundary conditions, in the computational domain express the physical realities:

- at the right boundary, (X=L), $\xi=\xi_L$, small variations of the variables are imposed [1]:

$$\frac{\partial\Psi}{\partial\xi}=\frac{\partial^2\Omega}{\partial\xi^2}=\frac{\partial\theta}{\partial\xi}=0. \qquad (7)$$

- at the left boundary of the domain, (X=0), $\xi=0$, the boundary conditions for the fluid and workpiece correspond to the entrance region:

$\theta=0;$ for the fluid and $\theta=1$ for the solid;

$$\Psi=\frac{U_\infty R^2}{2};\Omega=0; \qquad (8)$$

- at the solid/fluid interface, R=1, no slip and no penetration conditions are imposed for the velocity field while the continuity of temperature and heat flux are imposed for the temperature field

$\Psi=0;$

$$\Omega_{1,j}=\frac{1}{R_{surf}}-\frac{2\left[\Psi_{2,j}-\Psi_{1,j}-R_{surf}\cdot\Delta R\right]}{R_{surf}\cdot(\Delta R)^2}\ [1];$$

$$\theta_{N,j}=\frac{1/\Delta R_s}{1/\Delta R_s+K/\Delta R_f}\theta_{N-1,j}+$$
$$+\frac{K/\Delta R_f}{1/\Delta R_s+K/\Delta R_f}\theta_{N+1,j} \qquad (9)$$

where: the first subscript is the line of the grid and the second subscript is the column number; ΔR_f is the space on the radial network in the fluid domain: $R_W-1=(N-1)\Delta R_f$, while ΔR_s is the space on the radial network in the workpiece domain: $1=(N-1)\Delta R_s$; K is the ratio of the fluid and solid thermal conductivities.

- at the tunnel wall (the upper boundary), $R=R_W$, the no slip and no penetration conditions are imposed for the velocity field while a constant temperature case is considered for the temperature field:

$$\Psi = \frac{U_\infty R_w^2}{2}; \theta = 0;$$

$$\Omega_{N,j} = \frac{2\left[\Psi_{N-1,j} - \Psi_{N,j}\right]}{R_w \cdot \left(\Delta R_f\right)^2} [1]; \qquad (10)$$

- at the symmetry axis, R=0, the boundary condition for temperature express the symmetry approach:

$$\frac{\partial q}{\partial R} = 0. \qquad (11)$$

- Solving the governing equations (3÷6) with the boundary conditions (7÷11) allows the analysis of the velocity and temperature fields and determines the optimum design for the technological installation.

The finite differences method was used to solve the governing equations [NEA 06], [TAN 97]. The finite differences were centered in the computation domain while forward or backward finite differences were used for the boundary points. An iterative process solved the velocity field calculating the vorticity and the stream function fields and, further, the temperature field was found. Crank-Nicolson method was used for Ψ, θ and Ω solutions.

Having in view the cooling tunnel length, L_t, and the workpiece radius, r_s, a grid with 50 points on r direction (50 points for the solid and another 50 points for the fluid) was used; the number of points on the x direction varied between 50 and 100 depending on the maximum abscissa. A transient term was used for solving Ψ and Ω with a time step $\Delta\tau=10^{-7}$.

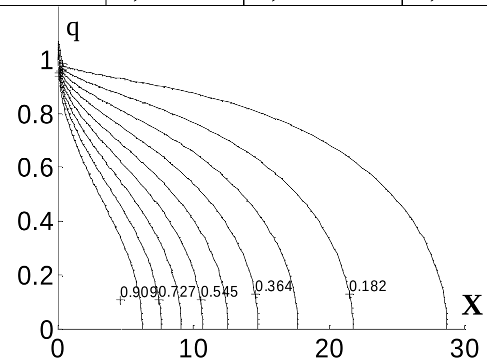
3. Results and discussions

Intense research was performed trying to determine the parameters that have a significant influence on the cooling process and on the industrial installation design: the tunnel radius, R_w, the tunnel wall temperature, T_w, the fluid/solid thermal conductivitie, K, the fluid velocity, U_∞, etc. The physical properties considered are presented by Table 1.

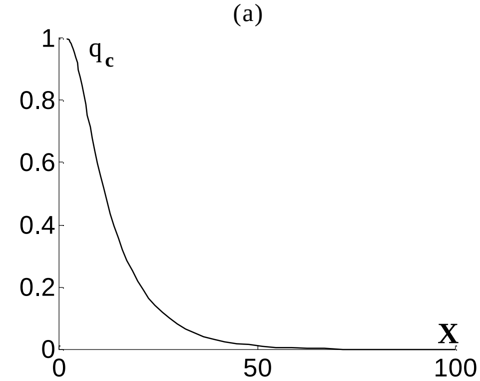
Figure 2 presents the isotherms (Fig. 2a) and the centerline temperature variation (Fig. 2b) for a glass fiber

Table 1. Physical properties

Solid/ liquid	Thermal Diffusivi -ty [cm^2/s]	Thermal Conductivi -ty [W/m/K]	Kinema tic vis-cosity [cm^2/s]
Teflon	0,001	0,23	-
Glass	0,0034	0,81	-
Alumin um	0,842	204	-
Water	0,001	0,59	0,01



(a)



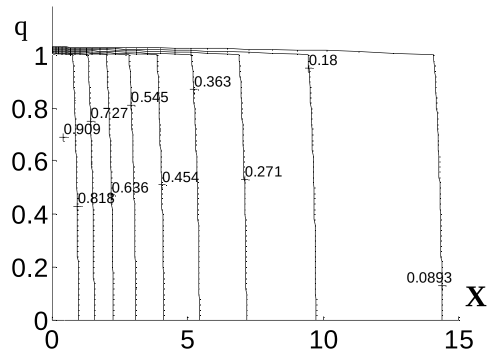
(b)

Fig. 2. Isotherms (a) and the centerline temperature variation (b) for a glass fiber extrusion. Pe=100, R_w=5, U_∞=2.

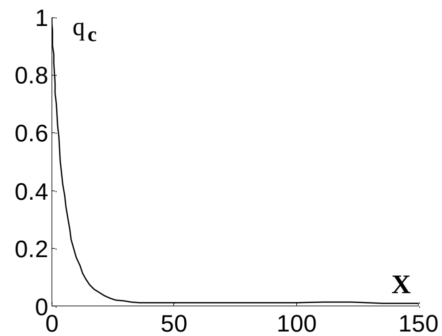
cooled with water with Pe=100, R_w=5 and U_∞=2. Figure 2b shows an optimum length of the cooling system, Lopt, a length defined by the abscissa where the centerline temperature becomes sufficiently smal (10^{-2}).

Figure 3 presents the isotherms (Fig. 3a) and the centerline temperature variation (Fig. 3b) for an aluminium wire extrusion case. The material properties play a major role in the temperature as can be noticed from the analysis of Fig. 2 and Fig. 3.

The research developed here showed that the Peclet numbe is the main parameter that influence the optimum cooling length for glass and teflon fibers, while the aluminium wires extrusion is influenced not only by the Peclet number but also by the radius of the tunnel, R_w.



(a)



(b)

Fig. 3. Isotherms (a) and the centerline temperature variation (b) for an aluminium wire extrusion. Pe=1, R_w=5, U_∞=2.

Figure 4 presents the optimum length variation as a function of Peclet number for a glass fiber case. As we can notice, the optimum length of the cooling system increases as Peclet number increases.

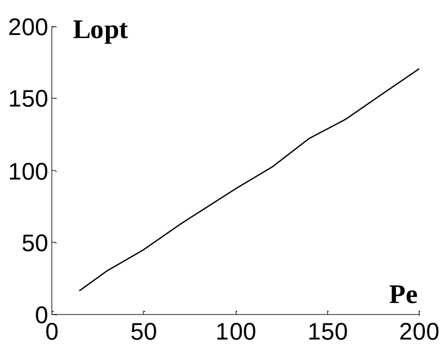


Fig. 4. Lopt - Pe variation for a glass fiber extrusion. R_w=5, U_∞=2.

The Lopt variation as a function of Peclet numer for the aluminium wire extrusion case is exemplified by Fig. 5 for R_w=5 and U_∞=2, while Fig. 6 presents the Lopt - R_w variation for Pe=1 and U_∞=2 for the same extrusion case. Both parameters determined a greater Lopt value as they increased.

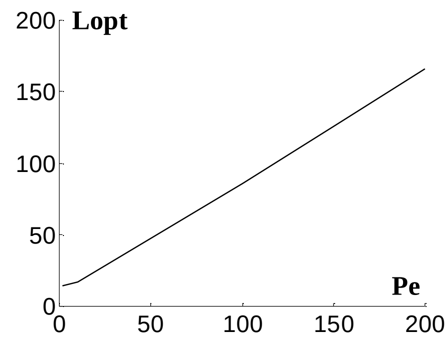


Fig. 5. Lopt - Pe variation for an aluminium wire extrusion. R_w=5, U_∞=2.

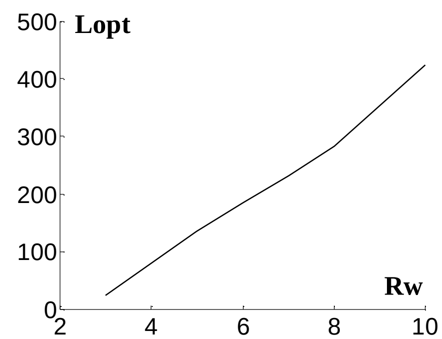


Fig. 6. Lopt - Rw variation for an aluminium wire extrusion. Pe=1, U_∞=2.

Table 2. Coefficients

Material	A	b	c
Teflon	1,120931	0,9462	-
Glass	1,328655	0,9093	-
Aluminum	0,119995	0,5565	2,4904

A general formula is proposed for Lopt for glass and teflon fibers (12a) as well as for aluminum wires (12b) extrusion:

$$Lopt = A \cdot Pe^{b}; \quad \text{(a)}$$

$$Lopt = A \cdot Pe^{b} \cdot R_w^{c}. \ \text{(b)} \qquad (12)$$

A, b and c are constants calculated using the minimum square method and their values, for the domain considered by Fig. 4÷ Fig. 6, are presented by Table 2. These formulas can be used in the design of the extrusion system.

4. Conclusions

- This work presents a numerical model of wires and fibers extrusion. The stationary case is considered for a cooling fluid flowing in the same direction as the workpiece, a cylindrical horizontal fiber/wire. A dimensionless vorticity-potential formulation is used for solution generality and for an easy interpretation of the results; Having in view the difference between the x and r dimensions, a non-uniform grid is used along the wire/fiber axis and the (ξ, R) computational domain is defined;

- The governing equations are solved using the finite differences method through an iterative process in an original analytical and numerical derivation;
- The optimum length of the cooling tunnel was defined and analyzed as a function of different material and process parameters. Peclet number, Pe, and the tunnel radius (for the aluminum extrusion), R_w, have the greater influence on the system optimum length variation;
- This analysis offers solutions for the optimum design of the wires/fibers extrusion system giving a design formula for the optimum length of the cooling system.

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Proiectarea instalației de extrudare a firelor/fibrelor

Rezumat

Aceasta lucrare analizează procesul de extrudare a firelor/fibrelor şi prezintă dependenta lungimii optime a canalului de răcire de parametrii de material şi de proces: numărul lui Peclet şi raza tunelului de răcire. Se prezintă îndrumări privind proiectarea sistemului de extrudare a firelor/fibrelor.

Die gestaltung von drähten und fasern extrusion systems

Zusammenfassung

Dieses papier analysiert die Drähte und fasern Extrusion und entwickelt eine Studie über die optimale Kühlung Kanal Länge Abhängigkeit von dem Material und Prozess-Parameter: die festen und flüssigen physikalischen Eigenschaften, die Draft/Glasfaser-Geschwindidkeit, die Kanal-Eigenschaften Abkühlung, etc. Dies hat zur Folge, Leitlinien sind für Draähte und fasern Extrusion systems Design.