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Chaotic Dynamics of Cutting Processes Applied to Reconfigurable Manufacturing Systems Control

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ABSTRACT

There are already known and enounced into dedicated literature the limits of classic theory concerning cutting processes stability. Starting from this aspect and also from the need of designing an intelligent system to control cutting stability, to enable full use of RMS technological system productivity resources, this paper is a first step to a new approach of cutting process dynamics, seen as chaotic phenomenon. By using Chaos Theory tools (Lyapunov exponent calculation, Poincare maps), a specific parameter variation, characterizing cutting process (the cutting force), is analyzed, in order to reveal a chaotic model of this process. Such a model will allow a better understanding of phenomena connected to cutting processes (in)stability, same time with imagining a simple and efficient tool to control the stability of cutting processes developed on RMS.
***Keywords**: cutting process, chaotic dynamics, reconfigurable, control.*

1. Limits of Cutting Process Stability Classic Theory

Even by considering recent scientific contributions to cutting process stability theory, a critic analysis reveals the following limits and unexplained things:

- Cutting process dynamics is not considered, although in cutting instability phenomenon it plays an essential role. Thus, it cannot be explained the dependence between cut material - chip shape – process stability (e.g. comparative stability between steel and bronze).
- There is no explanation for dependence between cutting speed and feed on one hand and stability limit, on the other hand, as this dependence can be experimentally observed (Fig.4) and nor to the relation between self-excited vibrations frequency and cutting speed, as we can also practically see in all cases.
- Instability phenomena appearing during a single cutting cycle cannot be justified, in this case regeneration phenomenon which stays at the base of current approach being inexistent.
- Current approach cannot explain why, as we can always see in practice, instability only appears when wave length of traces let by self-excited vibration on piece's surface has values between 0,5 and 12 mm and nor why at the middle of the interval the stability has a minimum level.
- Current theory cannot enable to find functioning point position referred to stability limit. More precisely, we cannot appreciate the reserve of stability existing at a given moment.
- To find stability limit, in the context of actual stability theory means to know system frequency characteristic. To obtain it supposes to follow a complex experimental plan. On the other hand, right in the moment when tool moves along worked piece generating line, frequency characteristic permanently changes. Thus, current theory doesn't offer the possibility of monitoring, in real time, technological system reserve of stability. This is the reason to intervene on the system only after it reached the unstable functioning domain. Briefly, any kind of stability prognosis is impossible, especially on-line, and that's why actual technological systems don't have a system to control stability.

Starting from these limits, on one hand and from the necessity of designing a tool able to be used to realize an efficient control of manufacturing processes stability, when a Reconfigurable Manufacturing System (RMS) is used, on the other hand, a possible solution could be a new approach to cutting process stability, by considering it a chaotic process.

Grounded on the new theory, an intelligent system to control cutting stability on RMS could be imagined, leading to a complete exploitation of technological system resources of productivity, by working with more intense cutting regimes, to the limit of stability domain. Thus, it will result both a maximisation of manufacturing processes efficiency and a superior quality of manufactured surfaces, by eliminating the risk of instability appearance.

2. Backgrounds of Considering the Cutting Process as Chaotic

It has been observed that all surfaces of solid objects are created by dynamic processes, be they mechanical, chemical or thermal. There is also evidence from measurements of surface topography that the apparently random displacements from the mean obey a scaling law [2] and experiments on fractured surfaces appear to show fractal scaling. These static properties of fractal surface topography have lead to propose that nonlinear dynamics may play a role in the machining or surface creation process.

The principal nonlinear effects on cutting dynamics include:

- Material constitutive relations;
- Tool-structure nonlinearities;
- Friction at the tool-chip interface;
- Loss of tool-worked piece contact;
- Influence of machine drive unit on the cutting flow velocity.

Studies of nonlinear phenomena in machine-tool operations involve three different approaches [1]:

(1) Measurement of nonlinear force-displacement behavior of cutting tools;

(2) Model-based studies of bifurcations using parameter variation;

(3) Time series analysis of dynamic data for system identification.

3. Cutting Process Chaotic Character Evaluation by Calculating Largest Lyapunov Exponent

For a dynamical system, sensitivity to initial conditions is quantified by the Lyapunov exponents. For example, consider two trajectories with nearby initial conditions on an attracting manifold. When the attractor is chaotic, the trajectories diverge, on average, at an exponential rate characterized by the largest Lyapunov exponent, [1].

This concept is also generalized for the spectrum of Lyapunov exponents, λ_i (i = 1, 2, ...,n), by considering a small n-dimensional sphere of initial conditions, where n is the number of equations (or, equivalently, the number of state variables) used to describe the system. As time t progresses, the sphere evolves into an ellipsoid whose principal axes expand (or contract) at rates given by Lyapunov exponents.

The presence of a positive exponent is sufficient for diagnosing chaos and represents local instability in a particular direction.

When the equations describing the dynamical system are available, one can calculate the entire Lyapunov spectrum. The approach involves numerically solving the system’s n equations for n+1 nearby initial conditions. In experimental settings, however, the equations of motion are usually unknown and this approach is not applicable.

For diverse experimental applications, a number of researchers have proposed algorithms that estimate the largest exponent.

The authors of this paper suggest the calculation of largest Lyapunov exponent in the case of time series including the values of cutting force measured during cutting process, to evaluate its potential chaotic character. The algorithm used to do this is presented below.

The first step of suggested approach involves reconstructing the attractor dynamics from a single time series, by using the method of delays, [2], to develop a fast and easily implemented algorithm. The reconstructed trajectory, X, can be expressed as a matrix where each row is a phase space vector,

$$X = \left[X_1 \, X_2 \ldots X_M \right]^T , \tag{1}$$

where X_i is the state of the system at discrete time i. For an N-point time series, $\{x_1, x_2, \ldots, x_N\}$, each X_i is given by

$$X_i = \left[x_i \, x_{i+J} \ldots x_{i+(m-1)J} \right], \tag{2}$$

where J is the reconstruction delay and m – the embedding dimension. Thus, X is an M x m matrix and the constants m, M, J and N are related as

$$M = N - (m-1)J . \tag{3}$$

After reconstructing the dynamics, the algorithm locates the nearest neighbor of each point on the trajectory. The nearest neighbor, $X_{\hat{j}}$, is found by searching for the point that minimizes the distance to the particular reference point X_j. This is expressed as

$$d_j(0) = \min_{X_{\hat{j}}} \left\| X_j - X_{\hat{j}} \right\| , \tag{4}$$

where $d_j(0)$ is the initial distance from the j^{th} point to its nearest neighbor and $\|..\|$ denotes the Euclidean norm. The condition that nearest neighbors should have a temporal separation greater then the mean period of the time series must also be considered.

$$\left|j-\hat{j}\right| > \text{mean period}. \qquad (5)$$

The largest Lyapunov exponent, λ_1, is then estimated as the mean rate of separation of the nearest neighbors.

The current approach is principally based on the work of Sato [3] which estimates λ_1 as

$$\lambda_1(i) = \frac{1}{i \cdot \Delta t} \cdot \frac{1}{M-i} \cdot \sum_{j=1}^{M-i} ln \frac{d_j(i)}{d_j(0)}, \qquad (6)$$

where Δt is the sampling period of time series and $d_j(i)$ is the distance between the j^{th} pair of nearest neighbors after i discrete-time steps.

Based on the upper presented algorithm, a dedicated soft was written in Turbo Pascal language. It calculates largest Lyapunov exponent in the case of a one-dimensional time series, starting from a file that contains an N-point time series.

The input data required to run the program are: the name of the file, J - the reconstruction delay, m – the embedding dimension and i – the number of discrete-time steps from relation (6). Time series number of points, N, is automatically found when reading the file including it.

The distances $d_j(0)$ are, first of all, calculated by using relation (4), also having in view the condition (5). Then, the distances $d_j(i)$ are similarly found and, finally, largest Lyapunov exponent is estimated $\lambda_1(i)$ by following (6).

According to above exposed considerations, the main sign that characterizes the chaotic character of a certain process is "the largest Lyapunov exponent". Thus, we must find a way to evaluate it, if we need to know if a certain cutting process is chaotic.

To do this job, we suggest the following procedure: first of all, a parameter among those who characterize the cutting process must be chosen; the most suitable parameter is the cutting force. Then, parameter (cutting force) values must be discretely measured for a certain time interval, by a Δt increment and recorded as a time series. Finally, the file including time series record can be analyzed by using the special dedicated soft [4], to calculate the largest Lyapunov exponent. A positive value of this exponent means a chaotic character of the analyzed cutting process.

The upper presented algorithm was applied to analyze turning processes. Cutting tests were realized on a SNB-360 turning machine, by manufacturing exterior cylindrical steel pieces. Different rotation speeds, different feeds and different cutting depths were successively used. Cutting force was discretely measured by a time increment $\Delta t = 10^{-4}$ s.

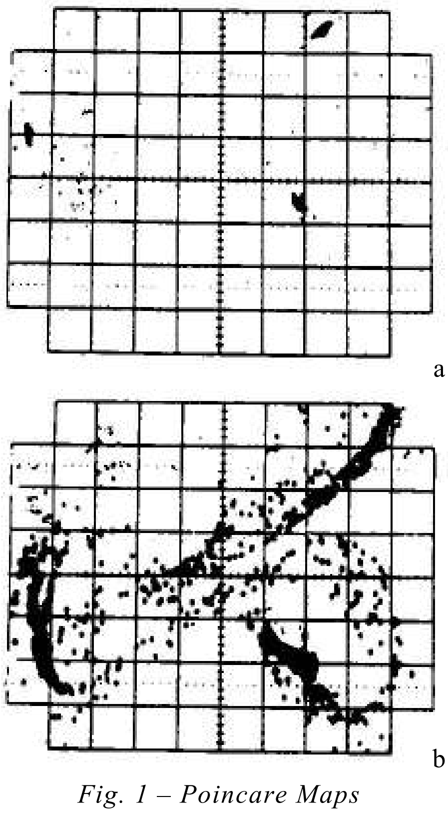
From the files including cutting force values, we chosen eight, and we have analyzed them by using the special dedicated soft. The values obtained for largest Lyapunov exponent, λ, in these cases are: 0.5953, 0.6450, 0.6565, 0.3210, 0.4546, 0.4347, 0.5738, and 0.5983. It means that considered cutting processes should be treated as chaotic.

4. Cutting Force Poincaré Map

In the mathematical study of dynamical systems, a map refers to a time-sampled data sequence $\{x(t_1), x(t_2), \ldots, x(t_n), \ldots, x(t_N)\}$, with the notation $x_n = x(t_n)$. A simple deterministic map is on in which the value of x_{n+1} can be determined from the values of x_n. This is often written in the form

$$x_{n+1} = f(x_n) \qquad (7)$$

The idea of a map can be generalized to more than one variable. Thus, x_n could represent a vector with M components, $x_n = \{Y1_n, Y2_n, \ldots, YM_n\}$ and equation (7) could represent a system of M equations.



a

b

Fig. 1 – Poincare Maps
a – periodically forced motion;
b – chaotic motion

For example, suppose we consider the motion of a particle as displayed in the phase plane, $(x(t), \dot{x}(t))$. However, if instead of looking at the motion continuously, we look only at the dynamics at discrete times, then the motion will appear as a sequence of dots in the phase plane (Fig. 1), [1].

If $x_n \equiv x(t_n)$ and $y_n \equiv \dot{x}(t_n)$, this sequence of points in the phase plane represents a two-dimensional map:

$$\begin{vmatrix} x_{n+1} = f(x_n, y_n); \\ y_{n+1} = g(x_n, y_n). \end{vmatrix} \quad (8)$$

When the sampling times t_n are chosen according to certain rules, this map is called a Poincaré map. When there is a driving motion of period *T*, a natural sampling rule for a Poincaré map is to choose $t_n = nT + \tau_0$. This allows one to distinguish between periodic motions and non-periodic motions.

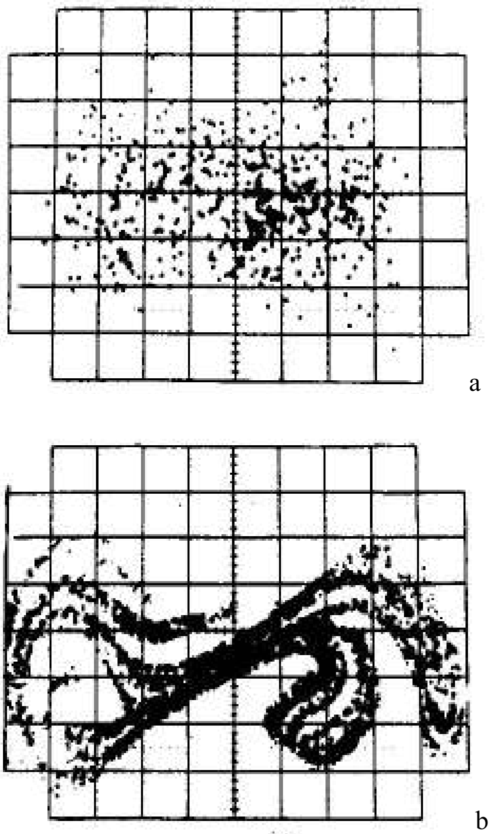


Fig. 2 – Poincaré Maps for Forced Vibrations Systems [1]
a – low damping case; b – higher damping case

If the Poincaré map does not consist of either a finite set of points (Fig. 1-a) or a closed orbit, the motion may be chaotic (Fig. 2). In undamped or lightly damped systems the Poincaré map of chaotic motion often appears as a cloud of unorganized points in the phase plane (Fig. 2-a), while in damped systems the map will sometimes appear as an infinite set of highly organized points, arranged as it can be seen in Fig. 2-b.

By assuming that there is a connection between cutting force variation and cutting process stability, we intended to confirm the chaotic character of cutting process dynamics by drawing Poincaré maps based on time series including cutting force, *F*, values. By using a dedicated soft to calculate points co-ordinates and AutoCad to realize the graphical representation of dependence between $\dot{F}$ and *F*, Poincaré maps from Fig. 3 were obtained.

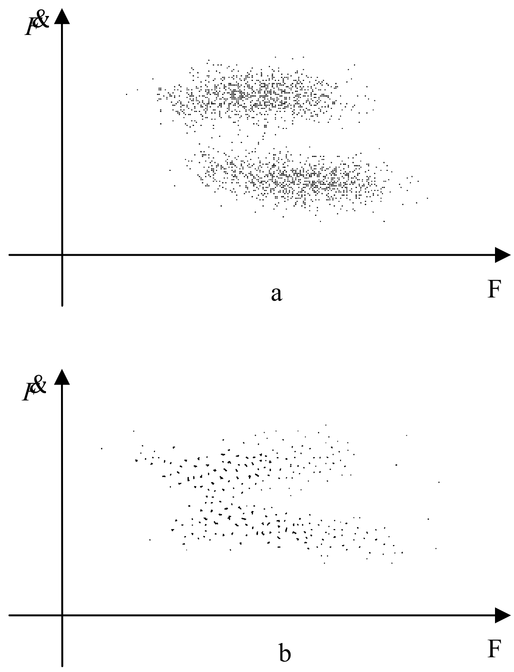


Fig. 3 - Poincaré Maps Drawn Based on Cutting Force Time Series

Poincaré map from Fig. 3-a was drawn by using a file including 4000 values of cutting force measured during a turning process while the other one (Fig. 3-b) is based on another file, with 2000 values of cutting force.

5. Conclusions

The results obtained by using the suggested algorithm to calculate the largest Lyapunov exponent in the case of cutting force time series are coming to consolidate the conviction that cutting processes should be considered as chaotic.

By analyzing Poincare maps presented in Fig. 3, the following conclusions can be highlighted:

- The aspect of the maps confirm the hypothesis that cutting process dynamics could be chaotic, in conjunction with positive values for greatest Lyapunov exponents found in the cases of same cutting force time series;
- The analyzed cutting processes present two states concerning its functioning stability, this fact being suggested by the existence of the two clearly delimitated sets of points.

The soft used to draw the maps add the points one by one; by following the way of obtaining the maps, it is obvious that the technological system oscillates between the two states (stable / unstable process), which means that we have to deal with a bifurcation point.

Starting from here, the necessity of developing the existing nonlinear models for complex dynamics in cutting materials or imagining new models of this type arises. There are yet problems to be clarified, as the limit-cycle behavior, unsteady chatter vibrations of the cutting tool, sub-critical Hopf bifurcations dynamics, cutting dynamics in non-regenerative processes, elasto-thermoplastic worked piece material instabilities, hysteretic effects in cutting dynamics, fracture processes in cutting of brittle materials, fracture effects in chip breakage etc.

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Dinamica haotică a proceselor de aşchiere în cazul sistemelor de fabricaţie reconfigurabile

Rezumat

Limitele teoriei clasice asupra stabilităţii proceselor de aşchiere sunt deja enunţate şi cunoscute. Pornind de la aceasta, precum şi de la necesitatea de a concepe un instrument inteligent care să poată fi utilizat la controlul stabilităţii aşchierii, în vederea exploatării depline a resurselor de productivitate ale sistemelor tehnologice reconfigurabile, lucrarea de faţă constituie un prim pas în direcţia unei noi abordări a dinamicii proceselor de aşchiere, considerate ca procese haotice. Prin utilizarea unor instrumente clasice ale Teoriei Haosului (determinarea exponenţilor Lyapunov, trasarea diagramelor Poincare), un parametru specific, ce caracterizează procesul de aşchiere (forţa de aşchiere), este analizat în vederea descoperirii unui model haotic al procesului. Un astfel de model va permite o mai bună înţelegere a fenomenelor legate de (in)stabilitatea proceselor de aşchiere, permiţând, în acelaşi timp, realizarea unui instrument simplu şi eficient pentru controlul stabilităţii proceselor de aşchiere desfăşurate pe sistemele de fabricaţie reconfigurabile.

La dynamique chaotique du processus d'enlevement des coupeaux dans le cas du sistèmes de fabrications réconfigurables

Resumé

Les limites de la théorie classique concernant la stabilité du processus d'usinage par enlèvement des coupeaux sont déjà connues. Si on commence d'ici et aussi de la nécessité d'inventer un instrument intelligent pour le contrôle de la stabilité du ce processus, pour entièrement exploiter les ressources de productivité des systèmes de fabrication reconfigurables, ce papier est un premier pas vers une nouvelle abordage de la dynamique du processus d'enlèvement des coupeaux, considéré comme chaotique. Par utilisant des instruments classiques de la Théorie du Chaos, un paramètre spécifique, en caractérisant le processus d'enlèvement des coupeaux (la force d'usinage) est analysé pour nous permettre de découvrir un modèle chaotique du processus.