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The Engendering of the Gevolvents Curves

Eugen GHIŢĂ

SUMMARY

The curves called gevolventes were defined relative recently, only included in some words of the authors of the presented paper. This naming was proposed regarding the similitudes of their engendering with the will-known curves evolvents (involutes). The difference is the fact that the engendering is done with a generatrix straight line that is rolling down, without sliding, on the interior of base curve line. The base curve line may by curve from an "extreme stellated polygon". The "evolvents" generated this way are always close curves, gyrating and that's why it is proposed for these the calling ***g- (gyrating) - evolvents!***

The gevolvents are extremely interesting curves as from the geometrical point of view as for the technical applications they can have. A series of these applications were developed and presented in the author's works.

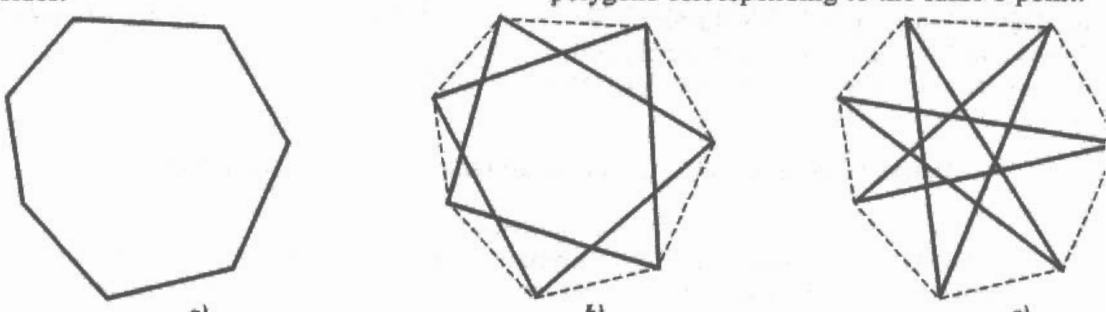
The paper propose to make then better known, presenting them using some new considerations that were realised regarding these ones. They hope that this way the specialists from different domains know them and eventually find new using for them.

Key words: *involute, evolvent, gevolvent, engendering curve.*

1. The Extreme Star Polygons

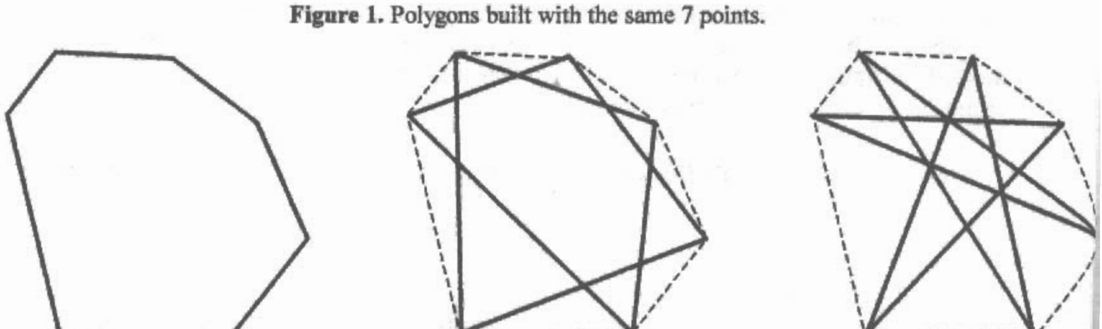
The notion *polygon* (word derived from Greek) defines a geometrical shape characterised of many (*poly-*) angles (*-gonos*). Each angle determines a peak. A polygon with ***n*** points has also ***n*** sides.

In concordance with this definition, with th same ***n*** points, with the same n points can b built ***(n-1)/2*** different polygon, if ***n*** is odd, ar respective ***(n-2)/2*** if ***n*** is a even number. In *fi 1* there are presented 3 polygons correspondir to the same 7 points, and in *fig. 2* the polygons corresponding to the same 8 point.



a) ***b)*** ***c)***

Figure 1. Polygons built with the same 7 points.



a) ***b)*** ***c)***

Figure 2. Polygons built with the same 8 points.

We call stellated polygons those ones whose sides intersect them selves (the cases *b* and *c*, fig. *1* and *2*). The polygon with the great number of intersection will be called "extre star polygon" (the cases c from *fig. 1* and 2).

In [1] there were made more relative considerations regarding the problem of defining the polygons. Here we will resume only the following theorem:

The sum of the angles of an extreme stellated polygon is:

- *180⁰* if the polygon has an ***n*** odd number of sides (***T1***);
- *360⁰* if the polygon has an ***n*** even number sides (***T2***).

We call a curvilinear extreme stellated polygon any geometrical construction having the sides as some arch of curve with the heads in the points of an extreme stellated polygon. More interesting for this paper, are the cases when all the sides of the curvilinear extreme stellated polygon, have no inflections, oriented to the interior of the polygon, see the *fig. 3.a*. And between these the most important are the ones that present the particularity that the curvilinear sides have in their points a common tangent, see the example from *fig. 3.b*.

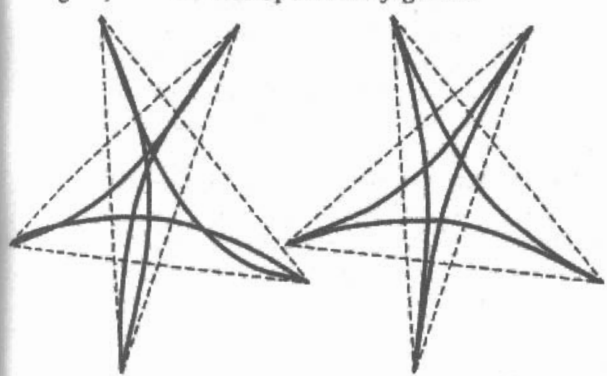


Fig. 3. Curvilinear extreme star polygon, with 5 sides.

2. The Definition of Gevolvents Curves

The evolvents are classical curves. In English they are more used involutes. By the definition, the evolvent are the generated curves as geometrical space described by a point belonging to a generating straight line that is rolling up on a curve, without sliding, called base – curve. The best known is the evolvent of the circle because this has, in technique, a lot of application. The evolvent is an opened curve, evaluating from the base curve to infinite, seeming with a spiral curve, *fig. 4*.

By definition the *gevolvents* (named shortly *curves **GE***) are generated as a geometrical place described by a place belonging to a straight generatrix that rules, without sliding, on a base curve line that is an extreme stellated polygon.

As a result of this different way of engendering, the curves always result closed! This particularity determined the adding at the word evolvent the initial g of the word gyrating, resulting the word gevolvent.

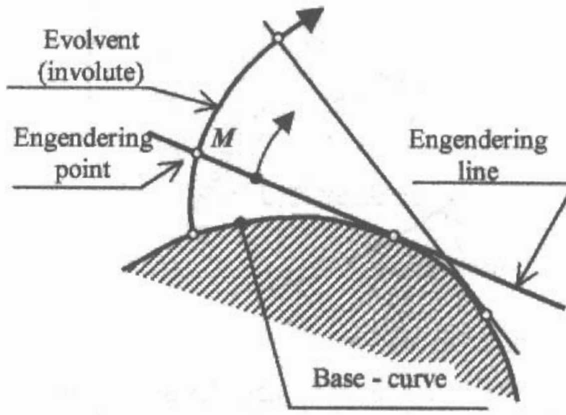


Fig. 4. The engendering of evolvent curve.

Observations:

a) When the ruling on one side gets to a point, the ruling continues on its appropriate side, fig. *5*;

b) If the generating point ***M*** is far away from the extreme stellated polygon (that is a curve base) and its sides don't intersect then the gevolvent result without inflections or point of coming back;

c) Regarding the position of the engendering point there can also result gevolvents having 2, 4, ... or maximum ***2n*** point of coming back, the gevolvent looking as a "tail of fish"; in the zones corresponding to some point;

d) When the engendering of a gevolvent has as base an extreme stellated polygon with an odd number of sides, then the resulted curve is always closed, also having the property *constant dimension*. I mean the normal straight line in any point of gevolvent, intersects the gevolvent for the second time still normally, and the distance between the 2 point of intersection is constant;

e) After the first ruling the straight generatrix, using all the sides of the polygon base, realises only one rotation with *180⁰* as a result of the property proved by theorem *T1*.Consequently, for the complete engendering there should be done even a second ruling, in order to realise a complete rotation with *360⁰*. So, the straight generatrix gets twice in the same position and the generatrix point ***M*** gets in the position ***M'*** (see the *fig. 5*);

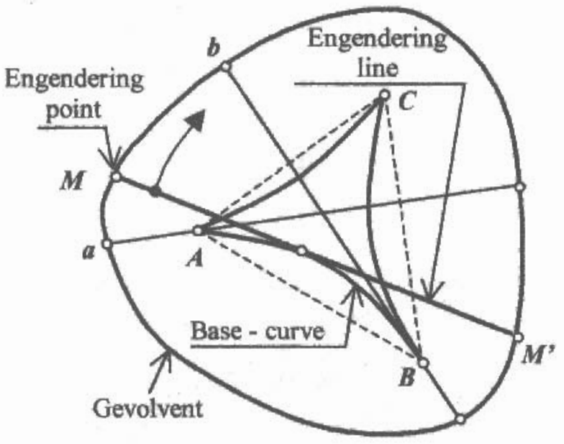


Fig. 5. The engendering of gevolvent curve

f) In the case of the engendering of some gevolvents using as a base an extreme stellated polygon with a even number of sides has to be completed supplementary the condition that

a-b+c-d=0,

where in a certain order. In this case the straight generatrix rules only once on all the sides of the polygon, because of the particularity resulted from the demonstration of the theorem *T2*. That's why the gevolvents results in this case, can't have the property "constant dimension". In *fig. 6* is represented as an example, the gevolvent built having as a curve base a curvilinear square, built with side's arches of circle;

g) The gevolvents represent the category of curves with the most numerous and interesting proprieties. From many of the proprieties here will be remarked only the one that, the circumference of all the gevolvents can be established with the same formula with that one of the circle, π.D, where D is the constant or average dimension $\frac{d_{max}+d_{min}}{2}$, see *fig. 6*.

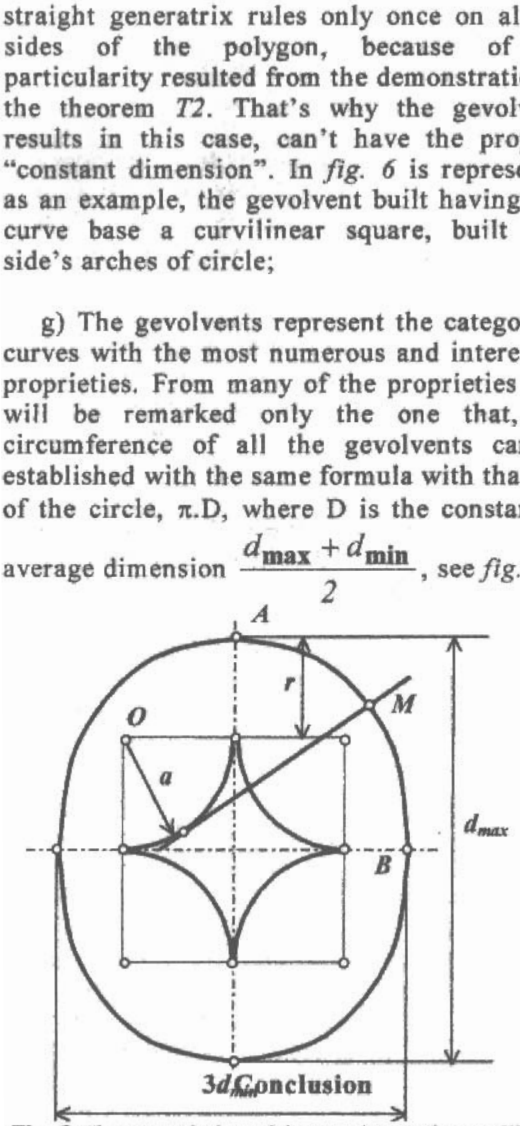


Fig. 6. The engendering of the gevolvent of a curvilinear

Conclusion

As it is intentioned, in this work there were presented only a few considerations regarding the engendering of the gevolvents, hoping to spread the knowledge referring to this new category of curves.

We also mention in these conclusions, that the gevolvents are very important as from theoretical point of view and also practically. The practical importance is amplified because of the particularities of this category of curves, from which will be sublimed the following:

- The gevolvents, generated using the same polygon base, have the property of being equidistant curves. This means that they there can be realised conjugated interior or exterior surfaces with play of constant measure;
- Because of the property "constant dimension" the gevolvent surfaces are easy to be measured and controlled, because it can be used universal instruments such as: calliper, micrometer, external gauge, etc.;
- Also, because of the property "constant dimension" the gevolvents surfaces may be used in the construction of mechanisms with cams, having the advantage of the fact they can be used in order to impart the movement in both sides, [4].

The researches that started in this domain [2], developed and presented a greater part in [1], but also in other papers, revealed the necessity of introduction in the specialised language of other new terms. So, for all the closed curves, with a periodic profile, was proposed the naming of polyforme curves. Respectively, also for the entire cylindrical, conical, helicoidally surface, whose section is limited by a polyforme curve, was proposed the naming polyforme surfaces. Between these, for a few categories of gevolvents where proposed the next particular names:

- Curves and polyconnected surfaces for the gevolvents of the regulated polygons (equilateral triangle, square, regulated pentagon, regulated hexagon, etc.);
- Curves and polyeccentrical surfaces for the gevolvents of some hypocycloids having a certain report I between the rays of rolling and base circle, *fig. 7*. We mention that these curves define the profiles used in the car construction industry for the removable assembly, as shaft – hub, wrongly named usually profiles as ***K*** or polygonal. These profiles are standardised in *Germany* and, probably in other countries.

Some other categories of gevolvents proved to be important too. They found out a series of technical applications in the construction of some mechanisms [2], [3] or for so called "unary gear" [1], having the transmission report *+1* and the pitch point to a infinite!

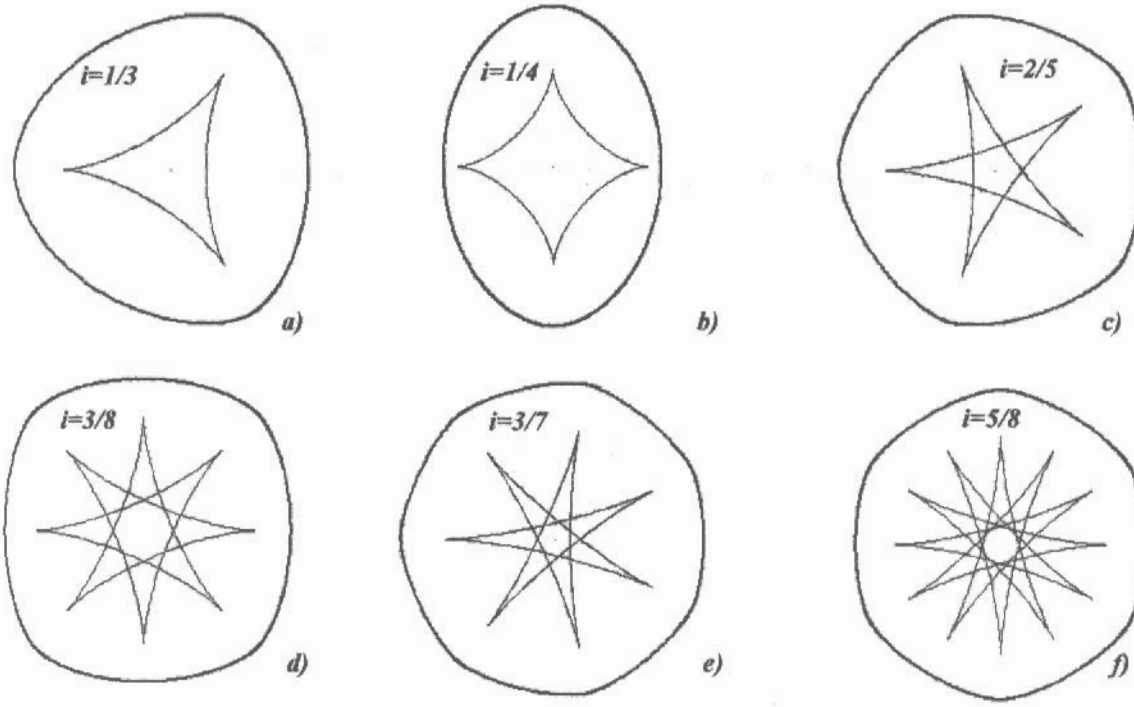


Fig. 7. The evolvents of some curvilinear regulated polygons.

We also mention that the theory of the engendering of the gevolvents determined the substantiation of new proceeds of processing by cutting, respectively by milling with plain milling cutter, face-milling cutter, toroidal - oscillating milling cutter or slotting by rolling (gear-shaped cutter).

Finally we mention more that the specialising realised during more than 20 years of researches in the domain of processing of some shaped surfaces permits us, at this moment, to attack also theoretical and practical any other research in this domain.

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Generarea curbelor Gevolventă

Rezumat. Curbele denumite gevolvente au fost definite relativ recent în diferitele lucrări ale autorului prezentei lucrări. Denumirea propusă a fost aleasă având în vedere similitudinile în ceea ce priveşte generarea ale deja cunoscutelor curbe denumite evolvente. Diferenţele în ceea ce priveşte generarea gevolventelor constau în aceea că aceste curbe se generează ca urmare a rulării unei drepte pe interiorul unui poligon, rezultând întotdeauna o curbă închisă. De aceea acestea au primit în denumire prefixul ***g*** de la iniţiala cuvântului *giratoriu*, devenind *gevolvente*. Aceste curbe s-au dovedit extrem de interesante atât din punctul de vedere al proprietăţilor geometrice cât şi al aplicaţiilor în tehnică, prezentate parţial în prezenta lucrare.

La création des courbes Gevolvents

Résume. Les courbes appelées les gevolventes étaient parent défini récemment, seulement inclus dans quelques mots des auteurs du papier présenté. Ceci appelant a été proposé concernant les similitudes de leur création avec les evolvents-connus de courbes (involutés). La différence est le fait que la création soit faite avec une ligne droite de génératrice en bas de la laquelle roule, sans glisser, sur l'intérieur de la ligne de base de courbe. La ligne de base de courbe peut par la courbe d'un extrême stellated le polygone. Les evolvents a produit de cette manière sont toujours les courbes étroites, tournant et des pourquoi on lui propose pour ces derniers le ***g*** appelant *giration* - devenant *gevolvents*! Les gevolvents sont les courbes extrêmement intéressantes comme du point de vue géométrique quant aux applications techniques ils peuvent avoir. Une série de ces applications a été développée et présentée dans les travaux des autors. Le papier propose de faire alors mieux connu, les présentant employant quelques nouvelles considérations qui ont été réalisées concernant ces derniers ceux. Ils espèrent que de cette façon les spécialistes de différents domaines les savent et trouvent par la suite nouvelle employer pour eux.