

THE CYCLOID TRAJECTORY METHOD APPLIED FOR DISCREETLY KNOWN PROFILES —II APPLICATIONS—

Ddr. ing. Virgil TEODOR, prof. dr. ing. Nicolae OANCEA,
 Universitatea "Dunărea de Jos" din Galați

ABSTRACT

The paper shows some examples for application of the cycloid trajectory theorem for the profiling of the rack-gear tool, gear-shaped cutter and rotary cutter tool. It's represented numerical examples and graphical illustrations of the trajectories family wrapping, for simple profiles engendering, known through the profile's points. For shown the result's correctness was realized the applications for some profiles as in case of the analytic known profiles (see [4], [5] and [6]).

1. Introduction

The algorithms for the known engendering proceedings (with rack-gear tool, gear-shaped cutter and rotary cutter) are the same as in the analytical known profiles.

It's evident, in this case, that the contact between the wrapping profile it is a punctiform contact. In following it's presented some applications of the new forms of the surfaces wrapping basic theorem for a variety of profiles and of the specifically engendering proceedings —the cycloid trajectory method.

2. Rack-gear tool engendering

It's presented applications assembly regarding the rack-gear engendering based on the stated theorem.

2.1. The rack-gear tool's profiling for squared profile

The first application, regarding also the figure 1, reported on the rack-gear tool's profiling for a squared crossing section shaft engendering.

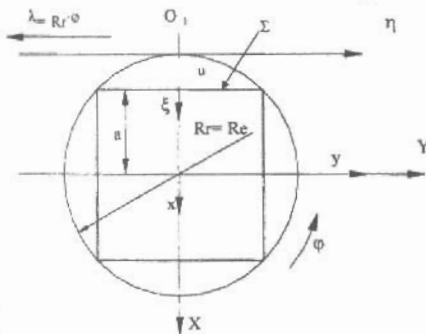


Figure 1. Squared shaft.

According with the proposed algorithm it's shown the crossing section shape of the shaft —

the profile's vortex, Σ — and the reference systems:

xy the fixed reference system, solidary with the blank's axis;

XY the mobile reference system, solidary with the profile's vortex;

$\xi\eta$ mobile reference system solidary with the rack-gear tool.

It's defined the parametrical equations for the shaft flank (the Σ profile for the ordinate profiles vortex):

$$M \begin{cases} X = -a; \\ Y = u, \end{cases} \quad (1)$$

with u variable parameter.

By the relative motion of the centroids associated spaces

$$\xi = \omega_3^T(\varphi) \cdot X - \begin{vmatrix} -Rr \\ -Rr \cdot \varphi \end{vmatrix} \quad (2)$$

it's determined the trajectory family:

$$(T_\Sigma)_\varphi \begin{cases} \xi = -a \cdot \cos \varphi - u \cdot \sin \varphi + Rr; \\ \eta = -a \cdot \sin \varphi + u \cdot \cos \varphi + Rr \cdot \varphi. \end{cases} \quad (3)$$

The trajectory family wrapper (3) is obtained, as shown in [1], associating with the $(T_\Sigma)_\varphi$ the wrapping condition in form

$$\frac{\xi'_u}{\xi'_\varphi} = \frac{\eta'_u}{\eta'_\varphi} \quad (4)$$

which, for this case, become

$$\frac{-\sin \varphi}{a \cdot \sin \varphi - u \cdot \cos \varphi} = \frac{\cos \varphi}{-a \cdot \cos \varphi - u \cdot \sin \varphi + Rr} \quad (5)$$

The (3) and (5) equation assembly represent the rack-gear profile reciprocally wrapping with the squared crossing section shaft.

The calculation is make by dividing the piece's profile into a number of equals spaces (in this

case 20) and the application of the same algorithms with the analytically known profile for each of these 20 resulted couple of points. Is considered the point's couple $M_i(x_i, y_i)$ and $M_{i+1}(x_{i+1}, y_{i+1})$. The length of this segment will be determined by relation

$$u = \sqrt{(x_{i+1} - x_i)^2 + (y_{i+1} - y_i)^2} \quad (6)$$

With notation

$$\alpha = \arctg\left(\frac{y_i - y_{i+1}}{x_i - x_{i+1}}\right) \quad (7)$$

is obtained the $\varphi(u)$ function which will have the form (from (5), (6) and (7))

$$\varphi(u) = -\arctg\left[\frac{\sqrt{1 - \left(\frac{u - x_i \cos \alpha - y_i \sin \alpha}{Rr}\right)^2}}{\frac{u - x_i \cos \alpha - y_i \sin \alpha}{Rr}}\right] - |\alpha| \quad (8)$$

Introducing the (6) and (8) equations in (3) equation is obtained the $(T_{\Sigma})_{\varphi}$ trajectory family:

$$\begin{cases} \xi_j = x_j \cos \varphi(u) - y_j \sin \varphi(u) - \\ - u \cos(\alpha + \varphi(u)) + Rr; \\ \eta = x_j \sin \varphi(u) + y_j \cos \varphi(u) - \\ - u \sin(\alpha + \varphi(u)) + Rr \cdot \varphi(u) \end{cases} \quad (9)$$

By comparative analysis of the values obtained for tool's coordinates is observed the perfect identity between these two sets of values.

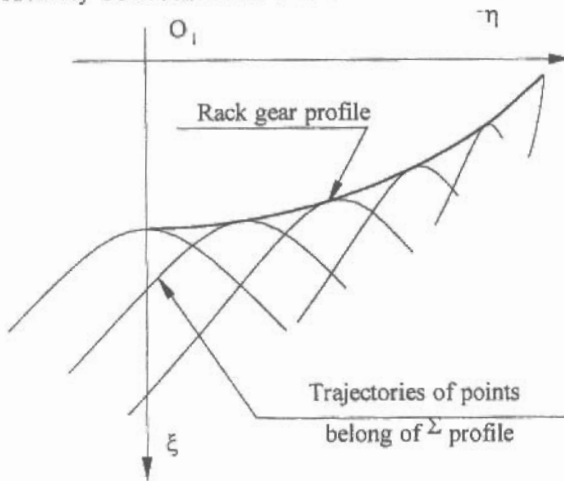


Figure 2 Trajectories of piece's points and wrapping of the rack-gear profile.

In figure 2 and table 1, are presented the shape and the coordinates of the rack-gear tool's profile, also the $(T_{\Sigma})_{\varphi}$ trajectory shape of point's belongs the Σ profile regarding the reference system associated with the C_2 centroid of the rack-gear tool, for case $Re=28,28$ mm, $a=20$ mm.

Table 1.

The points coordinates belongs of the piece's profile (x;y)	The points coordinates belongs of the tool's profile (analytical expression) (x;y)	The points coordinates belongs of the tool's profile (discreet expression) (x;y)
(-20; 17.994)	(1.4012; 20.660)	(1.4012;20.6603)
(-20; 15.995)	(2.7395; 18.8797)	(2.73952;18.8797)
(-20; 13.995)	(3.9744; 16.9048)	(3.97448;16.9048)
(-20; 11.996)	(5.0797; 14.7684)	(5.07975;14.7684)
(-20; 9.997)	(6.0373; 12.4994)	(6.0373;12.4994)
(-20; 7.997)	(6.8347; 10.1233)	(6.8347;10.1233)
(-20; 5.998)	(7.4628; 7.66383)	(7.4628;7.66383)
(-20; 3.998)	(7.91552;5.14286)	(7.91552;5.14286)
(-20; 1.999)	(8.18869;2.58146)	(8.18869;2.58146)
(-20; 0.0)	(8.28;0.0)	(8.28;0.0)
(-20; -1.999)	(8.1886; -2.58146)	(8.1886; -2.58146)
(-20; -3.998)	(7.91552; -5.1428)	(7.91552; -5.1428)
(-20; -5.998)	(7.4628; -7.66383)	(7.4628; -7.66383)
(-20; -7.997)	(6.8347; -10.1233)	(6.8347; -10.1233)
(-20; -9.997)	(6.0373; -12.4994)	(6.0373; -12.4994)
(-20; -11.996)	(5.0797; -14.7684)	(5.0797; -14.7684)
(-20; -13.995)	(3.9744; -16.9048)	(3.9744; -16.9048)
(-20; -15.995)	(2.7395; -18.8797)	(2.7395; -18.8797)
(-20; -17.994)	(1.4012; -20.6603)	(1.4012; -20.6603)
(-20; -19.994)	(-2e-005; -22.206)	(-2e-005; -22.206)

2.2. The rack-gear tool's profiling for spline shaft

In figure 3, is shown the crossing section of the spline shaft and the reference systems (see §2.1)

The parametrical equations of the flank have the form:

$$\Sigma \begin{cases} X = -u; \\ Y = b, \end{cases} \quad (10)$$

with "u" variable parameter.

In (2) motion, is generated the $(T_{\Sigma})_{\varphi}$ trajectory family:

$$\begin{cases} \xi = -u \cdot \cos \varphi - b \cdot \sin \varphi + Rr; \\ \eta = -u \cdot \sin \varphi + b \cdot \cos \varphi + Rr \cdot \varphi. \end{cases} \quad (11)$$

For this case, the wrapping condition (4) can be bring to form

$$\frac{-\cos \varphi}{u \cdot \sin \varphi - b \cdot \cos \varphi} = \frac{-\sin \varphi}{-u \cdot \cos \varphi - b \cdot \sin \varphi + Rr} \quad (12)$$

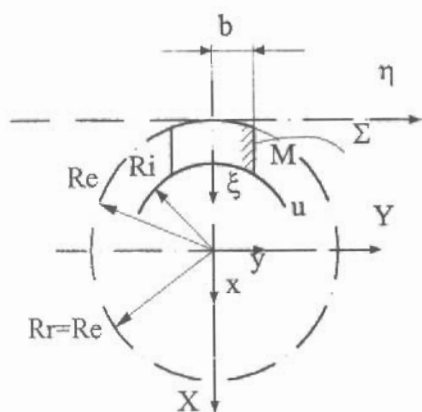


Figure 3. Spline shaft.

The (11), (12) equation assembly represent the rack-gear tool's profile reciprocally wrapping with the spline shaft flank.

The (9), (11) equation can be considered as trajectories of points belongs of Σ profile regarding the tool's reference system.

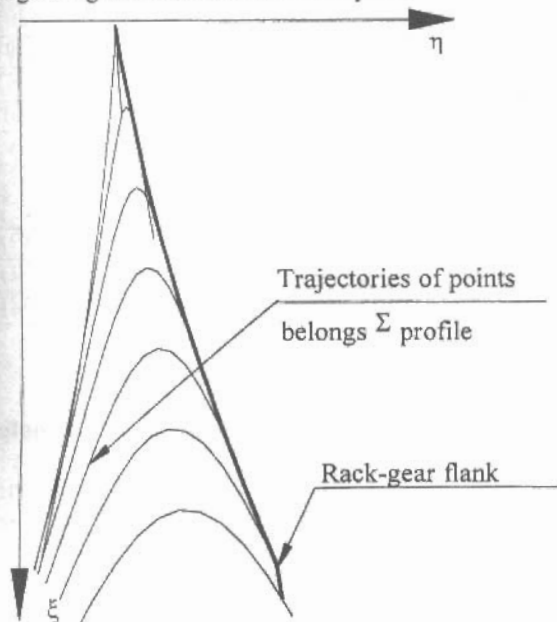


Figure 4. The $(T_{\Sigma})_{\phi}$ trajectories family wrapping.

In figure 4 and table 2, are shown the $(T_{\Sigma})_{\phi}$ trajectory (9) also the wrapping —rack-gear flank, for case: $R_i=24$ mm; $R_e=27$ mm; $b=4$ mm.

Table 2.

The points coordinates belongs of the piece's profile (x;y)	The points coordinates belongs of the tool's profile (analytical expression) (x;y)	The points coordinates belongs of the tool's profile (discreet expression) (x;y)
(-23.9681;4)	(3.8817;5.43402)	(3.8817;5.43402)
(-24.2719;4)	(3.4284;5.20602)	(3.4284;5.20599)
(-24.5756;4)	(2.9744;4.99207)	(2.974;4.9921)
(-24.8794;4)	(2.5207;4.79315)	(2.5207;4.79316)
(-25.1832;4)	(2.0688;4.61047)	(2.0688;4.61047)
(-25.487;4)	(1.6210;4.44551)	(1.6210;4.4455)
(-25.7907;4)	(1.1806;4.30015)	(1.1807;4.30017)
(-26.0945;4)	(0.75335;4.1769)	(0.75340;4.1769)
(-26.3983;4)	(0.3502;4.07966)	(0.3502;4.07966)
(-26.7021;4)	(-3e-005;4.0147)	(-3e-005;4.0147)

3. Gear-shaped cutter engendering

3.1. Gear-shaped cutter for squared bushing

It's shown an application of the algorithm for gear-shaped cutter profiling for squared profile bushing, see figure 5:

XY—mobile system solidary with squared bushing;

$\xi\eta$ —mobile system solidary with gear-shaped cutter.

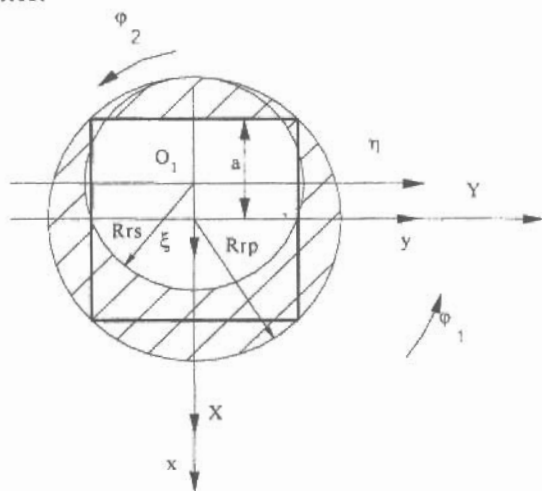


Figure 5. Square bushing profile.

The rolling centroid couple is composed by the circles with R_{rp} and R_{rs} radius.

The parametrical equations of the Σ profile are defined by (see figure 5)

$$\Sigma: X = -a; Y = u, \quad (13)$$

with "u" variable parameter.

From the relative motion

$$\xi = \omega_2(\phi_2) \cdot \left[\omega_3^T(\phi_1) \cdot X - a \right] \quad (14)$$

with

$$a = \begin{vmatrix} -A_{12} \\ 0 \end{vmatrix}; A_{12} = R_{rp} - R_{rs} \quad (15)$$

after replacements, results

$$\begin{pmatrix} \xi \\ \eta \end{pmatrix} = \begin{pmatrix} \cos \varphi_2 & \sin \varphi_2 \\ -\sin \varphi_2 & \cos \varphi_2 \end{pmatrix} \quad (16)$$

$$\begin{pmatrix} \cos \varphi_1 & -\sin \varphi_1 \\ \sin \varphi_1 & \cos \varphi_1 \end{pmatrix} \cdot \begin{pmatrix} -a \\ u \end{pmatrix} - \begin{pmatrix} -A_{12} \\ 0 \end{pmatrix}$$

As result is obtained the $(T_{\Sigma})_{\varphi}$ trajectory family belongs of the Σ profile in the relative motion of these regarding the $\xi\eta$ system:

$$(T_{\Sigma})_{\varphi I} \begin{cases} \xi = -a \cdot \cos(\varphi_1 - \varphi_2) - \\ -u \cdot \sin(\varphi_1 - \varphi_2) + A_{12} \cdot \cos \varphi_2; \\ \eta = -a \cdot \sin(\varphi_1 - \varphi_2) + \\ +u \cdot \cos(\varphi_1 - \varphi_2) - A_{12} \cdot \sin \varphi_2, \end{cases} \quad (17)$$

or, with notation,

$$\varphi_2 = i \cdot \varphi_1, i = \frac{Rrp}{Rrs} \quad (18)$$

$$(T_{\Sigma})_{\varphi I} \begin{cases} \xi = -a \cdot \cos(l-i)\varphi_1 - u \cdot \sin(l-i)\varphi_1 + \\ + A_{12} \cdot \cos(i\varphi_1); \\ \eta = -a \cdot \sin(l-i)\varphi_1 + u \cdot \cos(l-i)\varphi_1 - \\ - A_{12} \cdot \sin(i\varphi_1) \end{cases} \quad (19)$$

The wrapping of this trajectory family is determined by associating of these equations the (4) condition, where:

$$\begin{cases} \xi'_u = -\sin(l-i)\varphi_1; \\ \eta'_u = -\cos(l-i)\varphi_1; \\ \xi'_{\varphi_1} = a(l-i)\sin(l-i)\varphi_1 - \\ -u(l-i)\cos(l-i)\varphi_1 - iA_{12}\sin(i\varphi_1); \\ \eta'_{\varphi_1} = -a(l-i)\cos(l-i)\varphi_1 - \\ -u(l-i)\sin(l-i)\varphi_1 - iA_{12}\cos(i\varphi_1). \end{cases} \quad (20)$$

In practice, for the determination of the gear-shaped cutter profile, the piece's profile is divided into a number of segments and for each couple of points $M_i(x_i, y_i)$, $M_{i+1}(x_{i+1}, y_{i+1})$ the following algorithm is applied.

The length of the segment limited by these two points is calculated with the (6) equation.

From the (4) and (20) equations is determined the $\varphi(u)$ function, which for gear-shaped cutter will have the form.

$$\varphi(u) = \arctg \left[\frac{\sqrt{l - \left[\pm \frac{l \pm i}{iA_{12}} \cdot (-x_i \cos \alpha + y_i \sin \alpha + u) \right]^2}}{\left[\pm \frac{l \pm i}{iA_{12}} \cdot (-x_i \cos \alpha + y_i \sin \alpha + u) \right]} \right] + \alpha \quad (21)$$

Introducing the (6) and (21) equations in (19), is obtaining the $(T_{\Sigma})_{\varphi}$ trajectory family:

$$(T_{\Sigma i})_{\varphi} \begin{cases} \xi_i = x_i \cos[(l \pm i)\varphi(u)] - y_i \sin[(l \pm i)\varphi(u)] - \\ -u \cos[\alpha - (l \pm i)\varphi(u)] + A_{12} \cos(\pm i \cdot \varphi(u)); \\ \eta_i = x_i \sin[(l \pm i)\varphi(u)] + y_i \cos[(l \pm i)\varphi(u)] + \\ +u \sin[\alpha - (l \pm i)\varphi(u)] + A_{12} \sin(\pm i \cdot \varphi(u)). \end{cases} \quad (22)$$

In table 3 and figure 6, are shown the coordinates and the shape of the gear-shaped cutter profile and the $(T_{\Sigma})_{\varphi}$ trajectories of which

wrapping is the asked profile, for $a=40$ mm; $Rrs=30$ mm and $i=3/4$.

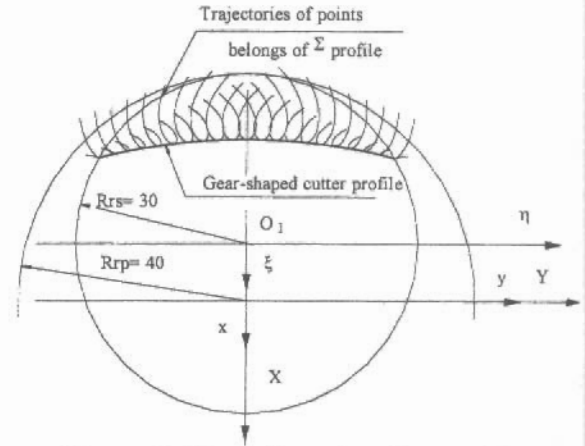


Figure 6. Gear-shaped cutter profile.

Table 3.

The points coordinates belongs of the piece's profile (x;y)	The points coordinates belongs of the tool's profile (analytical expression) (x;y)	The points coordinates belongs of the tool's profile (discreet expression) (x;y)
-28.28;22.62	(-16.258;20.620)	(-16.258;20.620)
-28.28;16.97	(-17.171;15.390)	(-17.1719;15.39)
-28.28;11.31	(-17.797;10.230)	(-17.7975;10.23)
-28.28;5.65	(-18.163;5.1071)	(-18.1636;5.10)
-28.2843;0.0	(-18.284;0.0)	(-18.2843;0.0)
-28.28;-5.656	(-18.163;-5.107)	(-18.163;-5.107)
-28.28;-11.31	(-17.797;-10.23)	(-17.797;-10.23)
-28.28;-16.97	(-17.171;-15.39)	(-17.171;-15.39)
-28.28;-22.62	(-16.258;-20.62)	(-16.258;-20.62)
-28.28;-28.28	(-15.0;-25.9808)	(-15.0;-25.9808)

3.2. Gear-shaped cutter for internal triangle splines

In figure 7, are shown the crossing section of the splines, the reference systems and the rolling centroids.

The spline flank, Σ , have the equations:

$$\Sigma \begin{cases} X = -R_0 - u \cdot \cos \varepsilon; \\ Y = u \cdot \sin \varepsilon, \end{cases} \quad (23)$$

with "u" variable parameter.

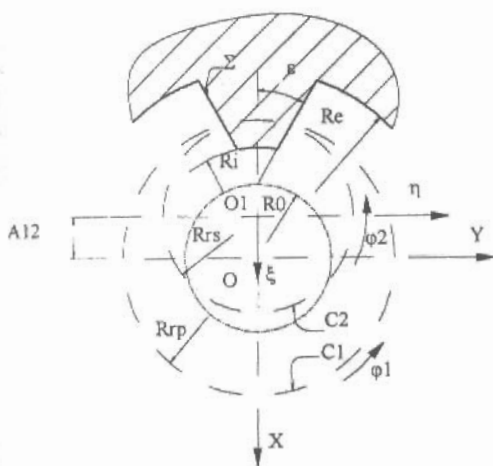


Figure 7. Internal spline.

In the relative motion (14), the Σ profile coordinates (23) determined the trajectory family:

$$(T_{\Sigma})_{\varphi I} \begin{cases} \xi = (-R_0 - u \cdot \cos \varepsilon) \cdot \cos(l-i)\varphi_I - \\ -u \cdot \sin \varepsilon \cdot \sin(l-i)\varphi_I + A_{12} \cdot \cos(i\varphi_I); \\ \eta = (-R_0 - u \cdot \cos \varepsilon) \cdot \sin(l-i)\varphi_I + \\ + u \cdot \sin \varepsilon \cdot \cos(l-i)\varphi_I - A_{12} \cdot \sin(i\varphi_I). \end{cases} \quad (24)$$

The wrapping condition (4) presume the partial derivative calculation:

$$\begin{cases} \xi'_u = -\cos \varepsilon \cdot \cos(l-i)\varphi_I - \sin \varepsilon \cdot \sin(l-i)\varphi_I \\ \eta'_u = -\cos \varepsilon \cdot \sin(l-i)\varphi_I + \sin \varepsilon \cdot \cos(l-i)\varphi_I \end{cases} \quad (25)$$

$$\begin{cases} \xi'_{\varphi I} = -(R_0 - u \cdot \cos \varepsilon)(l-i)\sin(l-i)\varphi_I - \\ -u(l-i)\sin \varepsilon \cos(l-i)\varphi_I - iA_{12} \sin(i\varphi_I); \\ \eta'_{\varphi I} = (R_0 - u \cos \varepsilon)(l-i)\cos(l-i)\varphi_I - \\ -u(l-i)\sin \varepsilon \sin(l-i)\varphi_I - iA_{12} \cos(i\varphi_I). \end{cases} \quad (26)$$

In figure 8 and table 4, are shown the trajectory shape (24), the wrapping profile (gear-shaped cutter profile) and those coordinates.

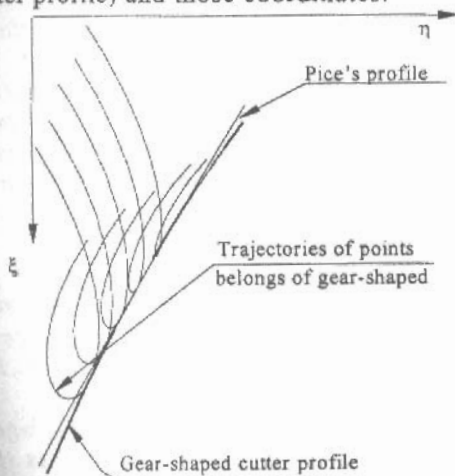


Figure 8. Gear-shaft cutter for internal spline

Table 4.

The points coordinates belongs of the piece's profile (x;y)	The points coordinates belongs of the tool's profile (analytical expression) (x;y)	The points coordinates belongs of the tool's profile (discreet expression) (x;y)
(-38.9;-0.61)	(-28.947;-0.593)	(-28.947;-0.593)
(-37.87;-1.2)	(-27.912;-1.152)	(-27.912;-1.152)
(-36.8;-1.84)	(-26.889;-1.682)	(-26.889;-1.682)
(-35.7;-2.45)	(-25.874;-2.187)	(-25.874;-2.187)
(-34.6;-3.07)	(-24.866;-2.669)	(-24.866;-2.669)
(-33.6;-3.68)	(-23.863;-3.131)	(-23.86;-3.131)
(-32.5;-4.29)	(-22.864;-3.575)	(-22.864;-3.575)
(-31.4;-4.91)	(-21.868;-4.001)	(-21.868;-4.001)
(-30.4;-5.52)	(-20.873;-4.411)	(-20.873;-4.411)
(-29.3;-6.14)	(-19.880;-4.806)	(-19.880;-4.806)

4. Rotary cutter engendering

It's proposed an example of application of the specific algorithm of rotary cutter engendering for profiling the tool, which generate a trapezoidal thread (see figure 9).

The process cinematic presume that is known the relative motion

$$\xi = \omega_3(\varphi)[X + a] \quad (27)$$

Regarding the parametrical equations of the Σ profile:

$$\begin{cases} X = -u \cdot \cos \alpha; \\ Y = u \cdot \sin \alpha, \end{cases} \quad (28)$$

with "u" variable parameter is determinated the trajectory family,

$$(T_{\Sigma})_{\varphi} \begin{cases} \xi = [-u \cdot \cos \alpha - Rrs] \cos \varphi + \\ + [u \cdot \sin \varphi - Rrs \cdot \varphi] \sin \varphi; \\ \eta = -[u \cdot \cos \alpha - Rrs] \sin \varphi + \\ + [u \cdot \sin \varphi - Rrs \cdot \varphi] \cos \varphi. \end{cases} \quad (29)$$

The wrapping condition (4) presume the partial derivative calculation:

$$\begin{cases} \xi'_u = \cos(\alpha + \varphi); \\ \eta'_u = \sin(\alpha + \varphi); \\ \xi'_{\varphi} = -[u \cdot \cos \alpha - Rrs] \sin \varphi + \\ + [u \cdot \sin \varphi - Rrs \varphi] \cos \varphi - Rrs \cdot \sin \varphi; \\ \eta'_{\varphi} = -[u \cdot \cos \alpha - Rrs] \cos \varphi - \\ - [u \cdot \sin \varphi - Rrs \varphi] \sin \varphi - Rrs \cdot \cos \varphi. \end{cases} \quad (30)$$

The (28), (4) and (30) equations assembly determine the trajectories family wrapping — the rotary cutter profile.

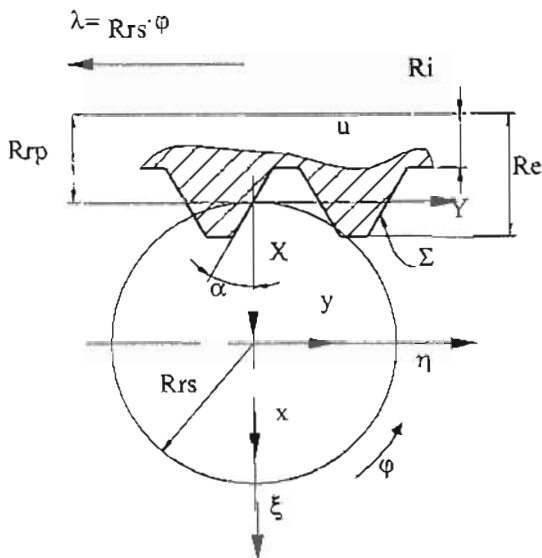


Figure 9. Trapezoidal thread —reference systems.

By application of the same algorithm as in anterior case, the $\varphi(u)$ function is obtained in form

$$\varphi(u) = \frac{(x_i \cos \alpha + y_i \sin \alpha + u)}{Rrs \sin \alpha} \quad (31)$$

The trajectory family wrapping will have the form:

$$(T_{\Sigma})_{\varphi} \begin{cases} \xi = x_i \cos \varphi(u) + y_i \sin \varphi(u) - \\ - u \cos(\alpha + \varphi(u)) - \\ - Rrs(\cos \varphi(u) + \varphi(u) \sin \varphi(u)); \\ \eta = x_i \sin \varphi(u) + y_i \cos \varphi(u) + \\ + u \sin(\alpha + \varphi(u)) + \\ + Rrs(\sin \varphi(u) - \varphi(u) \cos \varphi(u)). \end{cases} \quad (32)$$

In figure 10 and table 5, it's shown the $(T_{\Sigma})_{\varphi}$ trajectories (32), the rotary cutter profile and those coordinates.

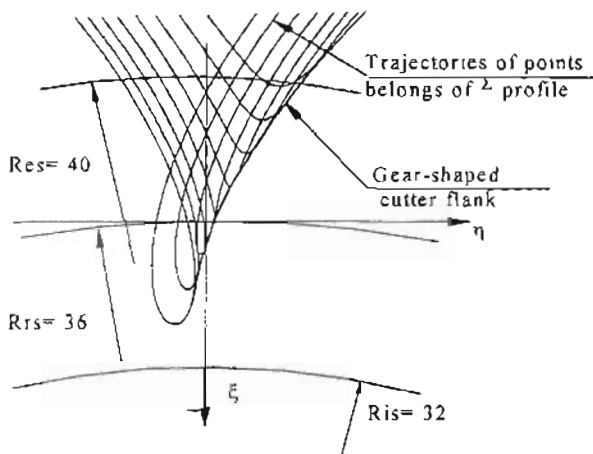


Figure 10. The gear-shaped cutter profile.

Table 5.

The points coordinates belongs of the piece's profile (x;y)	The points coordinates belongs of the tool's profile (analytical expression) (x;y)	The points coordinates belongs of the tool's profile (discreet expression) (x;y)
(-3.2;1.8475)	(-39.506;2.5643)	(-39.506;2.5643)
(-2.4;1.3856)	(-38.583;1.7809)	(-38.583;1.7809)
(-1.6;0.9238)	(-37.686;1.0958)	(-37.686;1.0958)
(-0.8;0.4619)	(-36.822;0.5039)	(-36.822;0.5039)
(0.0;0.0)	(-36.0;-4e-016)	(-36.0;-2.e-016)
(0.8;-0.4619)	(-35.22;-0.4218)	(-35.22;-0.4218)
(1.6;-0.9238)	(-34.50;-0.7679)	(-34.50;-0.7679)
(2.4;-1.3856)	(-33.84;-1.0448)	(-33.8401;-1.04)
(3.2;-1.8475)	(-33.241;-1.259)	(-33.241;-1.259)
(4.0;-2.3094)	(-32.710;-1.420)	(-32.710;-1.420)

5. Conclusions

The presented numerical examples attest the fact that the cycloid trajectory method can be successfully applied also if the engendering profile's surfaces vortex by the rolling method are known in discreet form.

More, is obviously the fact that these algorithms have a general character, and can be applied easily in case of analytical expressed profiles, which can easily be discretized through the variable incrementation.

6. References

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**Metoda traiectoriilor cicloidale aplicată
pentru profiluri cunoscute discret**

Rezumat

Lucrarea prezintă câteva aplicații ale teoremei traiectoriilor cicloidale pentru profilarea sculelor de tip cremalieră, cuțit-roată și cuțit rotativ. Sunt prezentate exemple numerice și ilustrații grafice ale înfășurătorii familiei de traiectorii, pentru generarea profilurilor simple, cunoscute prin puncte. Pentru a demonstra corectitudinea rezultatelor, aplicațiile au fost realizate pentru aceleași profiluri tratate analitic (vezi deasemenea [4], [5] și [6]).

**La méthode des trajectoires pour l'étude des
surfaces en enveloppement associé aux centroïdes en roulement.**

Resumé

En c'est ouvrages on présente quelques exemples d'utilisation du théoreme de la méthode des trajectoires pour profiler des outils type crémalier et couteau-roue. On présente des exemples numériques et des illustration graphiques des enveloppes d'un famille de trajectoires (voir aussi [4], [5] et [6]).