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THE CYCLOID TRAJECTORY METHOD APPLIED FOR DISCREETLY KNOWN PROFILES —I ALGORITHMS—

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ABSTRACT

The paper shows a method of calculations for the profiles of tools witch generates a desired profile witch is known through his points. This method is based on The Cycloid Trajectory Method for Wrapping Surfaces Associated of Rolling Centroid Study, [1]. The profile is indexing by his known points and these segments serves for the calculation of tool's profile [3], [4].

1. Introduction

It was shown the principle for generating by wrapping of the profiles associated of a couple of rolling centroids by the cycloid trajectory method [1]. For this was created the specific algorithm and was elaborated specialized software for the generating of compounded profiles formed from straight-line segments and circle's arcs. On this base were make applications for engendering with gear rack, gear-shaped cutter and rotary cutter.

The shown algorithms evidenced the quality of the method and the features of these.

The practice of tool's construction, based on the rolling method, exposes that are necessary, in many situations, to generate profiles which may leading at complicated equations, sometimes hard for use.

More, on these complex profiles, appears zones where the profiles are the results of a wrapping process with an another conjugated profile. This leads at complicated ways of profiles expressions for the piece's profiles.

Consequence of this fact is imposed the realization of a methodology based on a "discreet" representation of the profiles as a general way, capable to describe, by the cycloid trajectory method, a diversity of profiles, including complex analytical profiles or even non-analytical profiles.

2. The profiles discreet representation

We propose a discreet representation of profiles, based on the linearization of the curve's segments between two successive points of those, fig. 1, were M_i, M_{i+1}, M_{i+2} are successive points determinates on these curve.

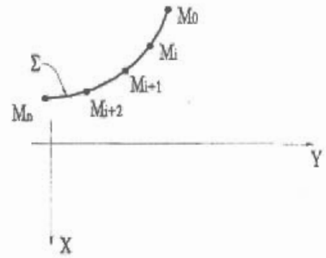

Figure 1. The curve segment's linearization.

If in XY reference system are defined the "discreet" coordinates of the profile, as a matrix with form

$$\Sigma = \left\| \begin{matrix} X_1 Y_1 \\ X_2 Y_2 \\ \vdots \\ X_i Y_i \\ X_{i+1} Y_{i+1} \\ \vdots \\ X_n Y_n \end{matrix} \right\| \quad (1)$$

where

$$\left| \sqrt{(X_{i+1} - X_i)^2 + (Y_{i+1} - Y_i)^2} \right| = \varepsilon$$

with enough small value for ε, then, according with the enunciated idea, the arc between $M_i(X_i, Y_i)$ and $M_{i+1}(X_{i+1}, Y_{i+1})$ points can be linearized as see in fig. 2.

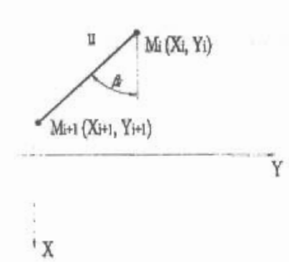

Figure 2. Replacement by straight segment.

Is defined, at length of M_iM_{i+1} segment the "u" variable so the segment's equations becomes:

$$X = X_i + u\cos\beta_i;\qquad Y = Y_i - u\sin\beta_i, \qquad (2)$$

with

$$tg\beta_i = \frac{|Y_{i+1} - Y_i|}{|X_{i+1} - X_i|}. \qquad (3)$$

With such representation of a profile, the application of the cycloidal trajectory family method becomes relatively simple.

3. Discreet represented profiles, generated by rolling with rack gear tool

It is known the process's cinematic of engendering with rack gear tool for a circular profile jointly with C_1 centroid, figure 3 and expressed in discreet form, see (1), (2) and (3).

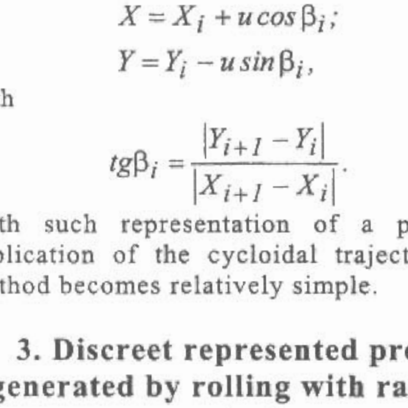

Figure 3. Rack-gear tool—reference systems.

The absolutes movements of the jointly systems with the wrapping profiles:

-of Σ blank,

$$x = \omega_3^T(\varphi_1)X; \qquad (4)$$

-of the rack gear tool,

$$\xi = x - a,\ a = \begin{Vmatrix} -Rrp \\ -Rrp\cdot\varphi_1 \end{Vmatrix}. \qquad (5)$$

Also, based of the (4) and (5) movements, is determinates the relative movement

$$\xi = \omega_3^T(\varphi_1)X - a, \qquad (6)$$

witch determinate the trajectory family

$$\begin{Vmatrix} \xi \\ \eta \end{Vmatrix} = \begin{Vmatrix} \cos\varphi_1 & -\sin\varphi_1 \\ \sin\varphi_1 & \cos\varphi_1 \end{Vmatrix} \cdot \begin{Vmatrix} X_i + u\cos\beta_i \\ Y_i - u\sin\beta_i \end{Vmatrix} - \begin{Vmatrix} -Rrp \\ -Rrp\cdot\varphi_1 \end{Vmatrix} \qquad (7)$$

or, after development:

$$T_{i\varphi}\ \begin{vmatrix} \xi = [X_i + u\cos\beta_i]\cos\varphi_i - \\ -[Y_i - u\sin\beta_i]\sin\varphi_i + Rrp; \\ \eta = [X_i + u\cos\beta_i]\cos\varphi_i - \\ -[Y_i - u\sin\beta_i]\sin\varphi_i + Rrp\cdot\varphi. \end{vmatrix} \qquad (8)$$

The $T_{i\varphi}$ trajectory family, for variable u between:

$$u_{min} = 0;\qquad u_{max} = \sqrt{(X_{i+1} - X_i)^2 + (Y_{i+1} - Y_i)^2}, \qquad (9)$$

limits, wrapping the rack gear tool's profile.

The determination of the rack gear tool's profile imposed the association with (8) equations (the trajectory family) of the wrapping condition

$$\frac{\xi'_u}{\xi'_\varphi} = \frac{\eta'_u}{\eta'_\varphi} \qquad (10)$$

which, considered the definitions:

$$\begin{aligned} \xi'_u &= \cos(\varphi + \beta_i); \\ \eta'_u &= \sin(\varphi - \beta_i); \\ \xi'_\varphi &= [-X_i\sin\varphi - Y_i\cos\varphi] - u\sin(\varphi - \beta_i); \\ \eta'_\varphi &= [X_i\cos\varphi - Y_i\sin\varphi] + u\cos(\varphi - \beta_i + Rr) \end{aligned} \qquad (11)$$

can become

$$u + X_i\cos\beta_i - Y_i\sin\beta_i + Rr\cos(\varphi - \beta_i) = 0. \qquad (12)$$

In this way, the equation assembly which represent the $T_{i\varphi}$ cycloid trajectory (8) and the specific wrapping condition (12) and the (1) definition of the Σ profile, represent the rack gear tool's profile.

4. Discreet represented profiles, generated by rolling with gear-shaped cutter

Similar with these the gear-shaped cutter's profiling, for Σ profile's engendering, discreet defined (see (1)), can be treated from the local liniarization method of the engendering profiles (see fig. 4 and (2) and (3) equations).

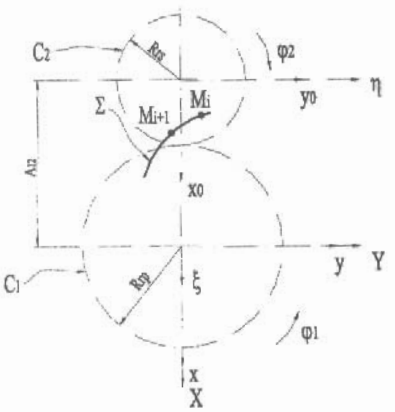

Figure 4. Gear-shaped cutter engendering.

In this way, regarding the engendering specific cinematic with gear-shaped cutter:

$$x=\omega_3^T(\varphi_1)X, \qquad (13)$$

$$x_0=\omega_3^T(-\varphi)\xi, \qquad (14)$$

$$x_0=x-a;\ a=\begin{Vmatrix}-A_{12}\\ 0\end{Vmatrix}, \qquad (15)$$

is defined the relative motion

$$\xi=\omega_3(-\varphi_2)\left[\omega_3^T(\varphi_1)X-a\right] \qquad (16)$$

through which is determined the Σ profile's point family trajectory regarding the reference system of the gear-shaped cutter tool —the cycloidal trajectory family,

$$\begin{Vmatrix}\xi\\ \eta\end{Vmatrix}=\begin{Vmatrix}\cos\varphi_2 & -\sin\varphi_2\\ \sin\varphi_2 & \cos\varphi_2\end{Vmatrix}\cdot\left[\begin{Vmatrix}\cos\varphi_2 & -\sin\varphi_2\\ \sin\varphi_2 & \cos\varphi_2\end{Vmatrix}\cdot\begin{Vmatrix}X_i+u\cos\beta_i\\ Y-u\sin\beta_i\end{Vmatrix}-\begin{Vmatrix}A_{12}\\ 0\end{Vmatrix}\right] \qquad (17)$$

or, after developments:

$$T_{i\varphi 1}\ \begin{cases}\xi=X_i\cos(1+i)\varphi_1-Y_i\sin(1+i)\varphi_1+\\ +u\cos[(1+i)\varphi_1-\beta_i]+A_{12}\cos i\varphi_1;\\ \eta=X_i\sin(1+i)\varphi_1+Y_i\cos\varphi_1+\\ +u\sin[(1+i)\varphi_1-\beta_i]+A_{12}\sin i\varphi_1,\end{cases} \qquad (18)$$

with $i=\dfrac{\varphi_2}{\varphi_1}$ transmission ratio.

Associating the (18) equations with the wrapping conditions (10) which can be bring in form

$$(1+i)X_i\cos\beta_i-(1+i)Y_i\sin\beta_i+(1+i)u+$$
$$+iA_{12}\cos(\varphi_1-\beta_i)=0, \qquad (19)$$

the assembly of these equations represent the gear-shaped cutter profile, obviously for "u" varying, incremental, between (9) conditions defined limits.

5. Discreet represented profiles, generated by rotary cutter

The reference systems and the movement parameters (λ and φ_1) are shown in figure 5.

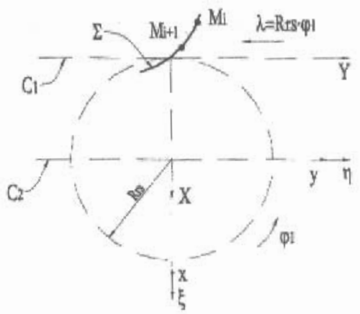

Figure 5. Rotary cutter engendering.

The relative motion of the centroids C_1 and C_2 is given by

$$\xi=\omega_3(\varphi_1)[X+a] \qquad (20)$$

In this way, regarding also (2), is determined the trajectory family:

$$T_{i\varphi 1}\ \begin{cases}\xi=X_i\cos\varphi_1+Y_i\sin\varphi_1+\\ +u\cos(\varphi_1+\beta_i)-Rrs\cos\varphi_1-\\ -Rrs\varphi_1\sin\varphi_1;\\ \eta=-X_i\sin\varphi_1+Y_i\cos\varphi_1-\\ -u\sin(\varphi_1+\beta_i)+Rrs\sin\varphi_1-\\ -Rrs\varphi_1\cos\varphi_1.\end{cases} \qquad (21)$$

The wrapping condition (see (10)) is

$$X_i\cos\beta_i-Y_i\sin\beta_i+u+Rrs\varphi_1\sin\beta_i=0. \qquad (22)$$

The (21) and (22) equations assembly represent for the discreet expressed profile (1), with "u" parameter variation limits (9), the wrapping profile of the rotary cutter's tooth flank.

6. Gearing lines

For these three analyzed situations, discreet expressed profiles associated with a couple of rolling centroids, C_1 and C_2, is defined gearing lines as the geometrical place of contacting points between these two conjugated profiles, in fixed reference system:

-rack-gear tool's and gear-shaped cutter engendering case,

$$\begin{aligned}&x=[X_i+u\cos\beta_i]\cos\varphi_1-\\ &-[Y_i-u\sin\beta_i]\sin\varphi_1;\\ &y=[X_i+u\cos\beta_i]\sin\varphi_1+\\ &+[Y_i-u\sin\beta_i]\cos\varphi_1;\\ &u+[X_i\cos\beta_i-Y_i\sin\beta_i]+\\ &+Rr\cos(\varphi-\beta_i)=0;\end{aligned} \qquad (23)$$

-rotary cutter engendering,

$$x = [X_i + u\cos\beta_i] + Rrs;$$
$$y = [X_i - u\sin\beta_i] - Rrs\varphi_1;$$
$$u + [X_i\cos\beta_i - Y_i\sin\beta_i] +$$
$$+ Rrs\varphi_1\sin\beta_i = 0. \quad (24)$$

7. Interference trajectories

Is possible, in some point of the profile defined by (1) matrix, to appear suddenly variation of the profile's tangent, or

$$tg\beta_i >> tg\beta_{i+1}$$
$$tg\beta_i << tg\beta_{i+1} \quad (25)$$

see also the (3) given definition.

In this way, it can considered that the "i" point with coordinates $[X_i, Y_i]$ represent a odd point on the discreet represented profile, and as result, in this point are defined two different normals, the wrapping conditions is undefined, the point describing in rapport with tool's reference system a interference trajectory.

So the reciprocally wrapping tool's profile with the blank's profile (1) result as a compound profile, determinate by points $M_1 \dots (M_i)$ and $(M_{i+1}) \dots M_n$, intersected by the interference trajectory generated by "i" point from the pieces profile, see fig. 6.

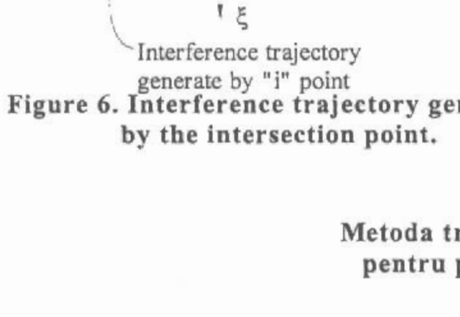

Figure 6. Interference trajectory generated by the intersection point.

8. Conclusions

The presented algorithms attest the fact that the cycloid trajectory method can be successfully applied also if the engendering profile's surfaces vortex by the rolling method are known in discreet form.

The wrapping conditions are simple and easy to use those make easy the method appliance.

More, is obviously the fact that these algorithms have a general character, and can be applied easily in case of analytical expressed profiles, which can easily be discreetized through the variable incrementation.

9. References

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Metoda traiectoriilor cicloidale aplicată pentru profiluri cunoscute în discret

Rezumat

Lucrarea prezintă o metodă pentru profilarea sculelor care generează un profil exprimat în discret. Metoda este bazată pe "Metoda traiectoriilor cicloidale pentru studiul suprafeţelor în înfăşurare asociate unor centroide in rulare" [1]. Profilul de generat este indexat prin punctele sale şi aceste segmente servesc pentru calculul profilului sculei.

La méthode de les trajectoires pour l'étude des surfaces en enveloppement associé aux centroides en roulement.

Resumé

On présente une modalité de profiler les outils qui genere un profile qui est exprimer en discret. La methode se basse sur "la méthode de les trajectories", pour l'étude de les surfaces en enveloppement associé aux centroids en roulement [1]. Le profile est indexer par son points et cettes segmentes sont utiliser pour la calculation du profile.