The Gear Hub Profiling for Machining Surfaces with Discreetly Expressed Surfaces

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Abstract: - They are know and used the various methods for the profiling of the gear hub reciprocally enwrapping with a profiles whirl, associated with a rolling centrodes.
In this paper, is proposed a profiling method of the gear hub primary peripheral surface, reciprocally enwrapping with an ordered profiles whirl based on the principles of helical motion decomposing and on the surfaces enwrapping theory. The proposed method is applied for surfaces known in discreetly form and approximated by 2nd or 3rd degree Bezier polynomials.
Based on a dedicated software are presented numerical examples and the results are shown versus results obtained by classical analytical methods, examples which show the reduced error level of proposed method.

Key-Words: - Gear hub, Bezier polynomial approximation, Discreetly expressed surfaces

1 Introduction

They are known modalities for gear hub primary peripheral surface profiling [1], [4], [5], [6], reciprocally enwrapping with an ordered surfaces whirl, associated with an axoid in rolling with a rack gear's axoid, common with the helical surface's rack of the gear hub.
Principled, the fundamentals theorems [4], [5], [6], of the surfaces enwrapping presume to know in analytically form the enwrapping surfaces, the contact conditions between the conjugated surfaces or analytical expression which associated with the generated surfaces family in the relative motions between the surfaces to be generated and the primary peripheral surfaces of tools, determine the form of the tools.
From some reasons may be important to know in discreetly form the surface to be generated [5].
These kinds of problem need a specific algorithm. The surface description, known in numerical form, by Bezier approximation polynomials, may constitute an alternative for the gear hub profiling when the generating precision is satisfactory. Obviously, this kind of solving is designated for the surfaces ordered whirl generation, firstly for the non-involute profiles, for which always appear the necessity of tool's profiling, when the tool's profile is not known.

2 Reference systems. Generating motion

In figure 1, is presented the system of rolling axoid: the surfaces to be generated whirl axoid; the rack-gear reciprocally enwrapping axoid; the gear hub primary peripheral surfaces axis position and the global motions of the reference systems associated with these axoids.

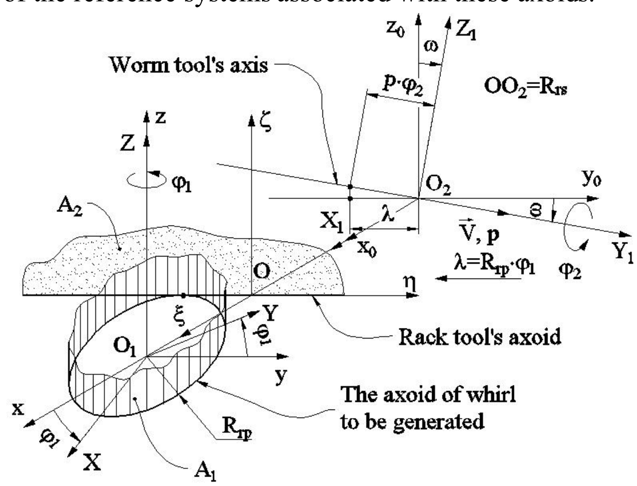

Fig. 1. The rolling axis and coordinate systems

They are defined the reference systems:
xyz is the global reference system with *z* revolving axis of the axoid associated with the whirl of surface to be generated;
$x_0y_0z_0$ — global reference system, with y_0 axis overlapped to the primary peripheral surface axis of the gear hub;
XYZ — relative reference system joined with the surface to be generated whirl, and axoid A_1;
$\xi\eta\zeta$ — relative reference system joined with the rack gear axis (in-plane surface overlapped with plane $\eta\zeta$), and axoid A_2;

$X_1Y_1Z_1$ — relative reference system associated with primary peripheral surface of gear hub.
Is known the kinematics of the generation process:

$$x = \omega_3^T(\varphi_1)X, \tag{1}$$

the A_1 axoid rotation, joined with the XYZ reference system, with φ angular parameter;

$$x = \xi + a,\ a = \begin{pmatrix} -R_{rp} \\ -\lambda \\ 0 \end{pmatrix}, \tag{2}$$

the A_2 axoid translation, joined with the $\xi\eta\zeta$ reference system and the λ movement parameter;

$$x_0 = \omega_2^T(\varphi_2)X_1, \tag{3}$$

the $X_1Y_1Z_1$ system rotation around the y_0 axis, with φ_2 angular parameter.
Also, are known the conditions:

$$\lambda = R_{rp}\cdot\varphi_1, \tag{4}$$

rolling condition of A_1 and A_2 axoid;

$$\lambda = p\cdot\varphi_2\cdot\cos\omega \tag{5}$$

dependency of gear hub primary peripheral surface, worm with known pitch (p — helical parameter) and the transformation between global reference systems:

$$x_0 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\kappa & -\sin\kappa \\ 0 & \sin\kappa & \cos\kappa \end{pmatrix}\cdot\left[\begin{pmatrix} x \\ y \\ z \end{pmatrix} - \begin{pmatrix} A_{12} \\ 0 \\ 0 \end{pmatrix}\right], \tag{6}$$

A_{12} distance between the axis of A_1 axoid and helical surface's axis $\vec{V}$.
In principle, the angular velocities for the revolution motions are even motions.
The relative motion of the reference system joined with A_1 axis of the surface to be generated, XYZ, regarding the reference system associated with the rack gear space, $\xi\eta\zeta$, is give by the transformation:

$$\xi = \omega_3^T(\varphi_1)X - a \tag{7}$$

with the current point's definition on the surface to be generated, as a cylindrical surface with generatrix parallel with the direction $Z(\vec{k})$:

$$\Sigma\left|\begin{aligned} &X = X(u); \\ &Y = Y(u); \\ &Z = t, \end{aligned}\right. \tag{8}$$

for u discreetly known variable with a reduced values number (3 or 4 points), as element of a complex profile which will be generate by enwrapping.
The crossing profile of the cylindrical surface (8) may be a straight lined segment, a circle arc, an involute arc etc.
The t parameter is measured along the cylindrical surface generatrix.
In following will be analyzed the problem of gear hub profiling using the expression of the cylindrical surface generatrix by a limited number of points. The generation kinematics is described by the transformation (7) and one of the fundamentals theorem of the enwrapping generation.

3 Rack gear surface form determination

From (7) and (8) are determined the surfaces family in the reference system of the rack-gear tool $\xi\eta\zeta$ with φ variable parameter:

$$\begin{pmatrix} \xi \\ \eta \\ \zeta \end{pmatrix} = \begin{pmatrix} \cos\varphi_1 & -\sin\varphi_1 & 0 \\ \sin\varphi_1 & \cos\varphi_1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\cdot\begin{pmatrix} X(u) \\ Y(u) \\ t \end{pmatrix} - \begin{pmatrix} -R_{rp} \\ -R_{rp}\cdot\varphi_1 \\ 0 \end{pmatrix} \tag{9}$$

which is associated with the enwrapping condition:

$$\left[X - X(u)\right]X'_u + \left[Y - Y(u)\right]Y'_u = 0 \tag{10}$$

where,

$$\begin{cases} X = R_{rp}\cos\varphi_1; \\ Y = R_{rp}\sin\varphi_1 \end{cases} \tag{11}$$

representing the "condition of normals".

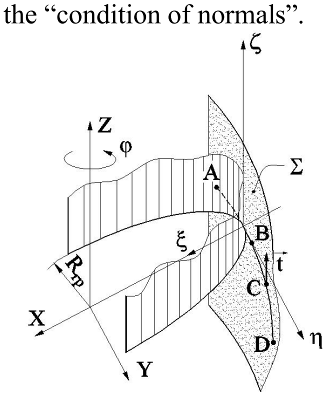

Fig. 2. The Σ surface of the whirl to be generated, known by four points on generatrix

In principle, the surfaces family described by equations (9) is on form:

$$\begin{aligned} \xi &= A_\xi\lambda^3 + 3\lambda^2(1-\lambda)B_\xi + \\ &+3\lambda(1-\lambda)^2 C_\xi + (1-\lambda)^3 D_\xi; \\ \eta &= A_\eta\lambda^3 + 3\lambda^2(1-\lambda)B_\eta + \\ &+3\lambda(1-\lambda)^2 C_\eta + (1-\lambda)^3 D_\eta. \end{aligned} \tag{12}$$

The identification of the A_ξ, B_ξ, C_ξ, D_ξ, A_η, B_η, C_η, D_η is presented in table 1, for a cylindrical surface with a circular generatrix regarding the example presented in subsection 5.2 and, similarly for another generatrix types.
Along the directrix of this surface type are defined a limited point number (3 or 4), which describe a Bezier polynomial replacing this curve.

Table 1. The rack-gear coefficients identification

λ		Points on tool's profile	Identification of the polynomials constants
0	$X_A = -R_{r_p}$ $Y_A = 0$	$\xi_A = -a + R_{r_p}$ $\eta_A = 0; \varphi_{1A} = 0$	$D_\xi = \xi_A$ $D_\eta = 0$
1/3	$X_B = -R_{r_p}$ $Y_B = \dfrac{a}{3}$	$\xi_B = -a\cos\varphi_{1B} - \dfrac{a}{3}\sin\varphi_{1B} + R_{r_p}$ $\eta_B = -a\sin\varphi_{1B} + \dfrac{a}{3}\cos\varphi_{1B} + R_{r_p}\cdot\varphi_{1B}$ $\varphi_{1B} = \arcsin\left(a/3R_{r_p}\right)$	$B_\xi = 3\xi_C - (3/2)\xi_B +$ $+(1/3)\xi_A - (5/6)\xi_D$ $B_\eta = 3\eta_C - (3/2)\eta_B +$ $+(1/3)\eta_A - (5/6)\eta_D$
2/3	$X_C = -R_{r_p}$ $Y_C = 2\dfrac{a}{3}$	$\xi_C = -a\cos\varphi_{1C} - \dfrac{2a}{3}\sin\varphi_{1C} + R_{r_p}$ $\eta_C = -a\sin\varphi_{1C} + \dfrac{2a}{3}\cos\varphi_{1c} + R_{r_p}\cdot\varphi_{1C}$ $\varphi_{1C} = \arcsin\left(2a/3R_{r_p}\right)$	$C_\xi = -(5/6)\xi_A + (1/3)\xi_D +$ $+3\xi_B - (3/2)\xi_C$ $C_\eta = -(5/6)\eta_A + (1/3)\eta_D +$ $+3\eta_B - (3/2)\eta_C$
1	$X_D = -R_{r_p}$ $Y_D = a$	$\xi_D = -a\cos\varphi_{1D} - a\sin\varphi_{1D} + R_{r_p}$ $\eta_D = -a\sin\varphi_{1D} + a\cos\varphi_{1D} + R_{r_p}\cdot\varphi_{1D}$ $\varphi_{1D} = \arcsin\left(a/R_{r_p}\right)$	$A_\xi = \xi_D$ $A_\eta = \eta_D$

By the identification of the polynomial coefficients is defined the discreetly form of the rack-gear reciprocally enwrapping with the whirl to be generated.

4 Gear Hub Primary Peripheral Surface Profiling

Being know the rack-gear flank surface is proposed the determination of the characteristically (the contact curve) at the contact with the future primary peripheral surface of the gear hub using the method of the helical movement decomposition [2].

Is accepted that the helical motion of the gear hub primary peripheral surface $\left(\vec{V}, p\right)$ is decomposing in a sum of equivalents motions: translation movement along the $\vec{t}$ direction of the unitary vector of the cylindrical surface generatrix and a revolution around $\vec{A}$ axis parallel with $\vec{V}$ and at distance

$$a = p \cdot \tan\theta \tag{13}$$

to the helical surface axis $\vec{V}$, see figure 3.

In this way, the S surface's characteristically curve, in the composed motion, don't depend to the motion component in which the surface is auto-generated, being fulfilled the identity:

$$\vec{N}_S \cdot \vec{t} \equiv 0, \tag{14}$$

(the normal at the S surface is always perpendicularly on the own generatrix), and the condition to determine the characteristically curve, in the helical motion $\vec{V}, p$, will depend only to the revolution around $\vec{A}$ axis.

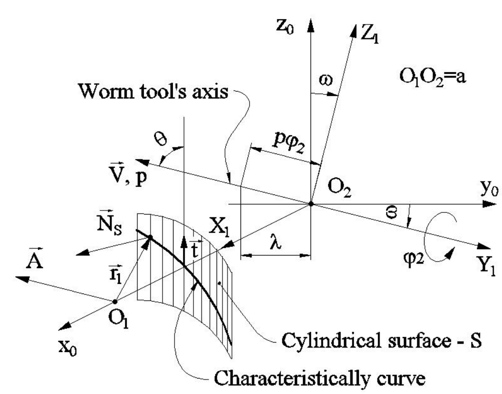

Fig. 3. Helical movements decomposition method. Reference systems

So, the characteristically curve of the S cylindrical surface is defined as the projection of the $\vec{A}$ axis on the S surface. The projection will be the geometric locus on

the S surface where the normals at this intersect the $\vec{A}$ axis.
They are defined, see figure 3:
- $\vec{A}$ axis, in the $x_0y_0z_0$ reference system,

$$\vec{A} = -\cos\omega \cdot \vec{j} + \sin\omega \cdot \vec{k}\ ; \quad (15)$$

- the normal at S surface, in form

$$\vec{N}_S = N_{x_0}\vec{i} + N_{y_0}\vec{j} + N_{z_0}\vec{k}\ , \quad (16)$$

with $N_{x_0}, N_{y_0}, N_{z_0}$ directrix parameters of the normal at S surface, approximated by an Bezier polynomial,

$$\vec{N}_S = \left(\frac{\partial x_0}{\partial \lambda}\vec{i} + \frac{\partial y_0}{\partial \lambda}\vec{j} + \frac{\partial z_0}{\partial \lambda}\vec{k}\right)\times \left(\frac{\partial x_0}{\partial t}\vec{i} + \frac{\partial y_0}{\partial t}\vec{j} + \frac{\partial z_0}{\partial t}\vec{k}\right); \quad (17)$$

- the $\vec{r}_1$ vector,

$$\vec{r}_1 = O_1O \cdot \vec{i} + \vec{r} \quad (18)$$

where $\vec{r}_1$ is the current point vector on the surface represented in the discreetly form, surface S.
From (12), result the coordinates transformation in the reference system $x_0y_0z_0$,

$$\begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix} = \begin{pmatrix} \xi \\ \eta \\ \zeta \end{pmatrix} - \begin{pmatrix} -R_{rs} \\ 0 \\ 0 \end{pmatrix} \quad (19)$$

leading at the S in the $x_0y_0z_0$ reference system:

$$\left| \begin{aligned} & x_0 = A_\xi\lambda^3 + 3\lambda^2(1-\lambda)B_\xi + \\ & +3\lambda(1-\lambda)^2 C_\xi + (1-\lambda)^3 D_\xi + R_{rs}; \\ & y_0 = A_\eta\lambda^3 + 3\lambda^2(1-\lambda)B_\eta + \\ & +3\lambda(1-\lambda)^2 C_\eta + (1-\lambda)^3 D_\eta; \\ & z_0 = t, \end{aligned} \right. \quad (20)$$

with R_{rs} pitch radius of the gear hub.
The ω parameter value is determined from condition that the helix on the cylinder with R_{rs} radius to be parallel with the rack gear generatrix cylindrical flank, see figure 4:

$$\tan\omega = \frac{2\pi p}{2\pi R_{rs}} = \frac{p}{R_{rs}} \quad (21)$$

with p helical parameter of the primary peripheral surface of gear hub.
In this way, the condition to determine the characteristically curve become:

$$\left(\vec{A}, \vec{N}_S, \vec{r}_1\right) = 0\ . \quad (22)$$

In principle, the (22) equation represent a dependency between λ and t variable parameters: $q(\lambda, t) = 0$, with $0 \le \lambda \le 1$.
The (20) and (22) equations assembly represent a geometric locus on the S surface with significance of the S surface's characteristically curve.
The couple of parametrical values λ and t for which is satisfied the (22) condition by replacing the rack-gear flank determine the matrix

$$C_S = \begin{pmatrix} X_{O_1} & Y_{O_1} & Z_{O_1} \\ \vdots & \vdots & \vdots \\ X_{O_i} & Y_{O_i} & Z_{O_i} \\ \vdots & \vdots & \vdots \\ X_{O_n} & Y_{O_n} & Z_{O_n} \end{pmatrix}^T \quad (23)$$

representing the C_S characteristically curve's coordinates.

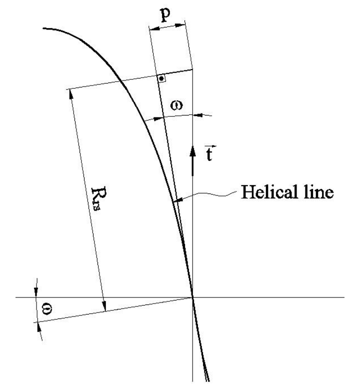

Fig. 4. Unfold of the helical line

Is made the coordinate transformation from $X_0Y_0Z_0$ reference system to a system $X_1Y_1Z_1$ with Y_1 axis overlapped with the gear hub tool's axis, see figure 1:

$$X_1 = \omega_1^T(\omega) \cdot X_0 \quad (24)$$

so, in the helical movement

$$X_1 = \omega_2(\varphi_2) \cdot \omega_1^T(\omega) \cdot X_{O_i} + p\varphi_2\vec{j}\ , \quad (25)$$

of the C_s curve it arrive at form:

$$\Pi \left| \begin{aligned} & X_1 = X_1\left(X_{O_1}, Y_{O_1}, Z_{O_1}, \varphi_2\right); \\ & Y_1 = Y_1\left(X_{O_1}, Y_{O_1}, Z_{O_1}, \varphi_2\right); \\ & Z_1 = Z_1\left(X_{O_1}, Y_{O_1}, Z_{O_1}, \varphi_2\right), \end{aligned} \right. \quad (26)$$

representing the gear hub's primary peripheral surface equations.
Associating with Π surface the condition

$$Z_1 = 0\ , \quad (27)$$

is obtained the axial section of the gear hub, in principle, in form:

$$\Pi_A = \left| \begin{aligned} & X_1 = X_1\left(X_{O_1}, Y_{O_1}, Z_{O_1}, \varphi_2\right); \\ & Y_1 = Y_1\left(X_{O_1}, Y_{O_1}, Z_{O_1}, \varphi_2\right); \end{aligned} \right. \quad (28)$$

with φ_2 corresponding to the axial section $Z_1 = 0$.

5 Numerical examples

5.1 Shaft with square crossing section

As first example is proposed a shaft with square crossing section, see figure 5.

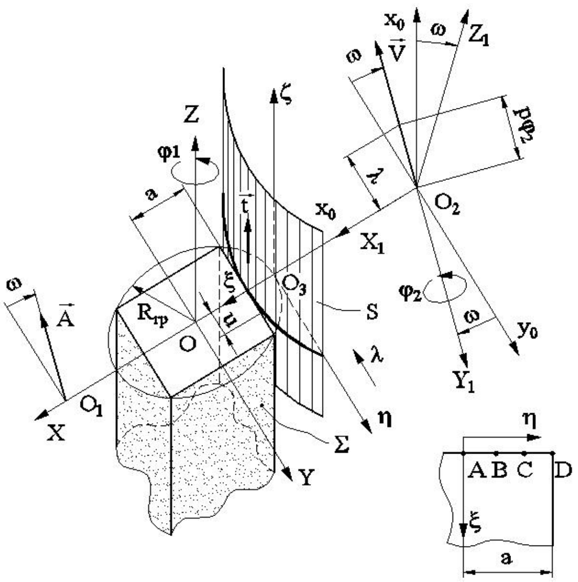

Fig. 5. Shaft's crossing section and reference systems

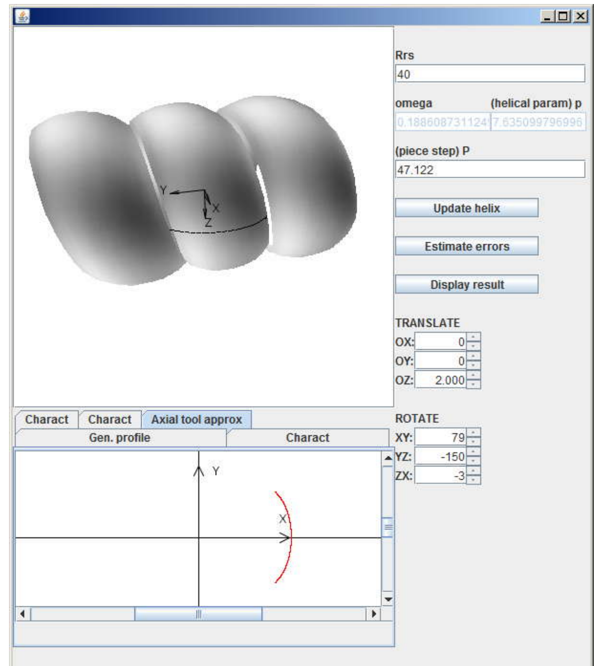

Fig. 6. Software for gear hub profiling; axial section and characteristically curve

The Σ surface to be generated has equations:

$$\Sigma\begin{vmatrix} X=-a; \\ Y=u; \\ Z=t, \end{vmatrix} \tag{29}$$

with a side of square, u and t variables.

The Σ surface equations in the reference system of the rack-gear tools will be:

$$\begin{pmatrix}\xi\\ \eta\\ \zeta\end{pmatrix}=\begin{pmatrix}\cos\varphi_1 & -\sin\varphi_1 & 0\\ \sin\varphi_1 & \cos\varphi_1 & 0\\ 0 & 0 & 1\end{pmatrix}\cdot\begin{pmatrix}-a\\ u\\ t\end{pmatrix}-\begin{pmatrix}-R_{rp}\\ -R_{rp}\varphi_1\\ 0\end{pmatrix}, \tag{30}$$

or

$$\begin{vmatrix} \xi=-a\cos\varphi_1-u\sin\varphi_1+R_{rp}; \\ \eta=-a\sin\varphi_1+u\cos\varphi_1+R_{rp}\varphi_1; \\ \zeta=t. \end{vmatrix} \tag{31}$$

In figure 6, is presented a screenshot of the software used to profile the gear hub; also the characteristically curve and axial section are presented there.

In table 2, are presented the gear hub profile coordinates calculated for the tool which profile the shaft with dimensions: R_{rp}=30 mm; a= 42.42 mm; R_{rs}=40 mm; p_e=47.1 mm (helical pitch), versus the results obtained by using an analytical method (Gohman [4]), for the same surface. The approximate Bezier polynomial for the surface's generatrix (30), is a 3[rd] degree polynomial. The error level of the gear hub is $2\cdot10^{-2}$ mm regarding the theoretically profile.

Table 2. Gear hub profile coordinates

λ	Approximated profile [mm]		Real profile [mm]		Error [mm]
	X_1	Y_1	X_1	Y_1	
0,000	**48,790**	**0,000**	**48,790**	**0,000**	**0,000**
	48,769	1,303	48,780	1,303	0,012
	⋮	⋮	⋮	⋮	⋮
	48,029	7,775	48,035	7,775	0,006
0,333	**47.852**	**8,618**	**47,855**	**8,617**	**0,003**
	47.753	9,050	47,754	9,049	0,001
	⋮	⋮	⋮	⋮	⋮
	45,177	16,430	45,172	16,429	0,005
0,666	**44,994**	**16,805**	**44,991**	**16,806**	**0,003**
	44,589	17,596	44,589	17,598	0,002
	⋮	⋮	⋮	⋮	⋮
	40,917	22,999	40,925	23,007	0,011
1,000	**40,022**	**23,967**	**40,041**	**23,948**	**0,026**

5.2 Chain wheel

In following, is presented another example of the proposed algorithm for the helical surface reciprocally enwrapping with a chain wheel.

The dimensional elements of the wheel are: p_w=15 mm (wheel pitch); dividing radius, R_{rs}=36.0727 mm; z=15 teeth, A [-35; 0]; B [-36.25; 2.16]; C [-40; 8.66]; R_1=2.5 mm; R_2=7.5 mm; R_{rs}=40 mm; ω=0.3843 rad; helical parameter p=16.1807 mm.

The wheel's profile is presented in figure 7, and the gear hub coordinates are given in table 3 and 4.

The error level regarding the tool's profile determined by analytical methods is on level $1 \cdot 10^{-2}$ [mm].

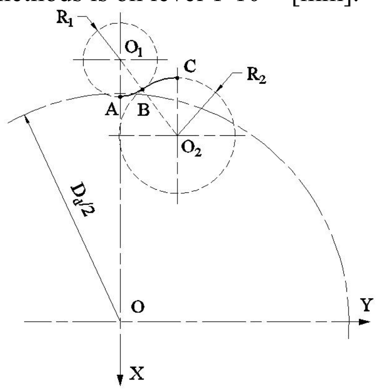

Fig. 7. Chain wheel's profile

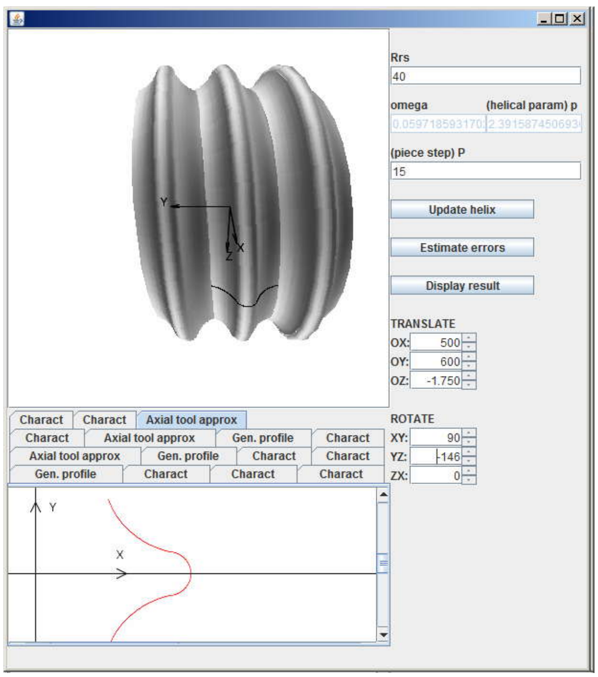

Fig. 8. Software screenshot; axial section and characteristically curve

Table 3. Gear hub profile for AB arc

λ	Approximated profile [mm]		Theoretically profile [mm]		Error [mm]
	X_1	Y_1	X_1	Y_1	
0,000	**41,000**	**0,001**	**41,000**	**0,002**	**0,001**
	⋮	⋮	⋮	⋮	⋮
	40,884	0,834	40,885	0,835	0,001
0,333	**40,856**	**0,924**	**40,857**	**0,926**	**0,001**
	40,841	0,971	40,841	0,972	0,001
	⋮	⋮	⋮	⋮	⋮
	40,427	1,734	40,426	1,734	0,001
0,666	**40,397**	**1,769**	**40,397**	**1,769**	**0,001**
	40,333	1,840	40,334	1,840	0,001
	⋮	⋮	⋮	⋮	⋮
	39,809	2,198	39,809	2,199	0,001
1,000	**39,703**	**2,224**	**39,705**	**2,222**	**0,003**

Table 3. Gear hub profile for BC arc

λ	Approximated profile [mm]		Theoretically profile [mm]		Error [mm]
	X_1	Y_1	X_1	Y_1	
0,000	**39,692**	**2,224**	**39,692**	**2,219**	**0,005**
	⋮	⋮	⋮	⋮	⋮
	37,700	3,324	37,698	3,329	0,004
0,333	**37,511**	**3,496**	**37,511**	**3,499**	**0,003**
	⋮	⋮	⋮	⋮	⋮
	36,034	5,430	36,034	5,427	0,002
0,666	**35,976**	**5,538**	**35,975**	**5,535**	**0,003**
	⋮	⋮	⋮	⋮	⋮
1,000	**35,104**	**7,926**	**35,105**	**7,924**	**0,002**

In figure 8, is presented a screenshot of the software used for gear hub profiling

4 Conclusion

The profiling method of the gear hub which generates an ordered profiles whirl is based on the principles of the helical motion decomposition.

The method use the Bezier approximation polynomials for the rack-gear reciprocally enwrapping with profiles whirl and is rigorous enough, for usually profiles, as is proved by the presented numerical examples.

The method has a general character and the method precision may be increased by the increasing the approximate Bezier polynomials degree, for characteristically curves approximation.

For the polynomials with reduced degree are defined pre-calculated forms of the coefficients.

Acknowledgement

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References:

[1] Ball, R.S., *A Treatise on the Theory of Screws*, Cambridge University Press, Cambridge, 1990;

[2] Lukshin, V.S., *Theory of Screw Surfaces in Cutting Tool Design*, Mashinstroyenie, Moscov, 1968;

[3] Radzevich, S.P., *Kinematic Geometry of Surface Machining*, CRC Press, London, ISBN 978-1-4200-6340-0, 2008;

[4] Litvin, F.C. *Theory of Gearing*, Reference Publication, 1212, NASA, Scientific and Technical Information Division, Washington, D.C, 1989;

[5] Oancea, N., *Surface generation through Winding*, Galați, 2004, ISBN 973-627-206-4;

[6] Teodor, V., Dima, M., Oancea, N., *Profilarea sculelor prin metode analitice*, Galați, 2005, ISBN 973-627-333-4.